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SCHOOL OF CIVIL ENGINEERING

INDIANA DEPARTMENT OF HIGHWAYS

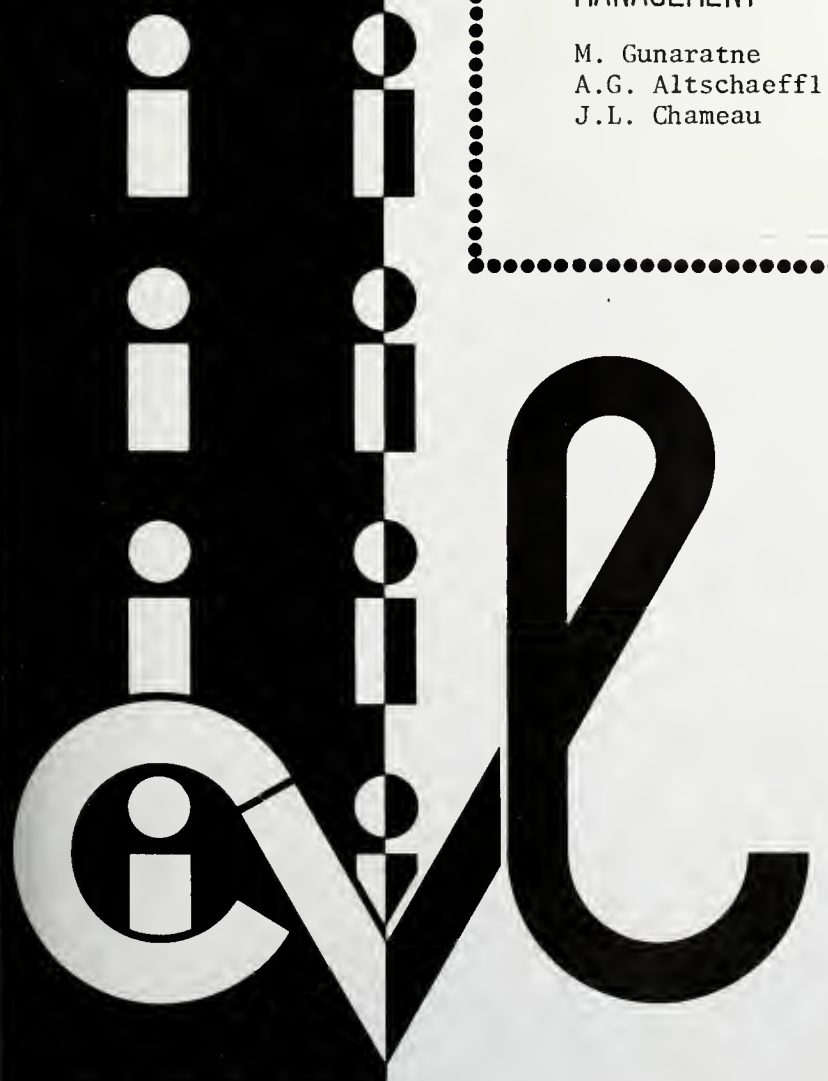
JOINT HIGHWAY RESEARCH PROJECT

Interim Report

FHWA/IN/JHRP-84/18

THE USE OF FUZZY SETS MATHEMATICS IN PAVEMENT EVALUATION AND MANAGEMENT

M. Gunaratne
A.G. Altschaeffl
J.L. Chameau



PURDUE UNIVERSITY

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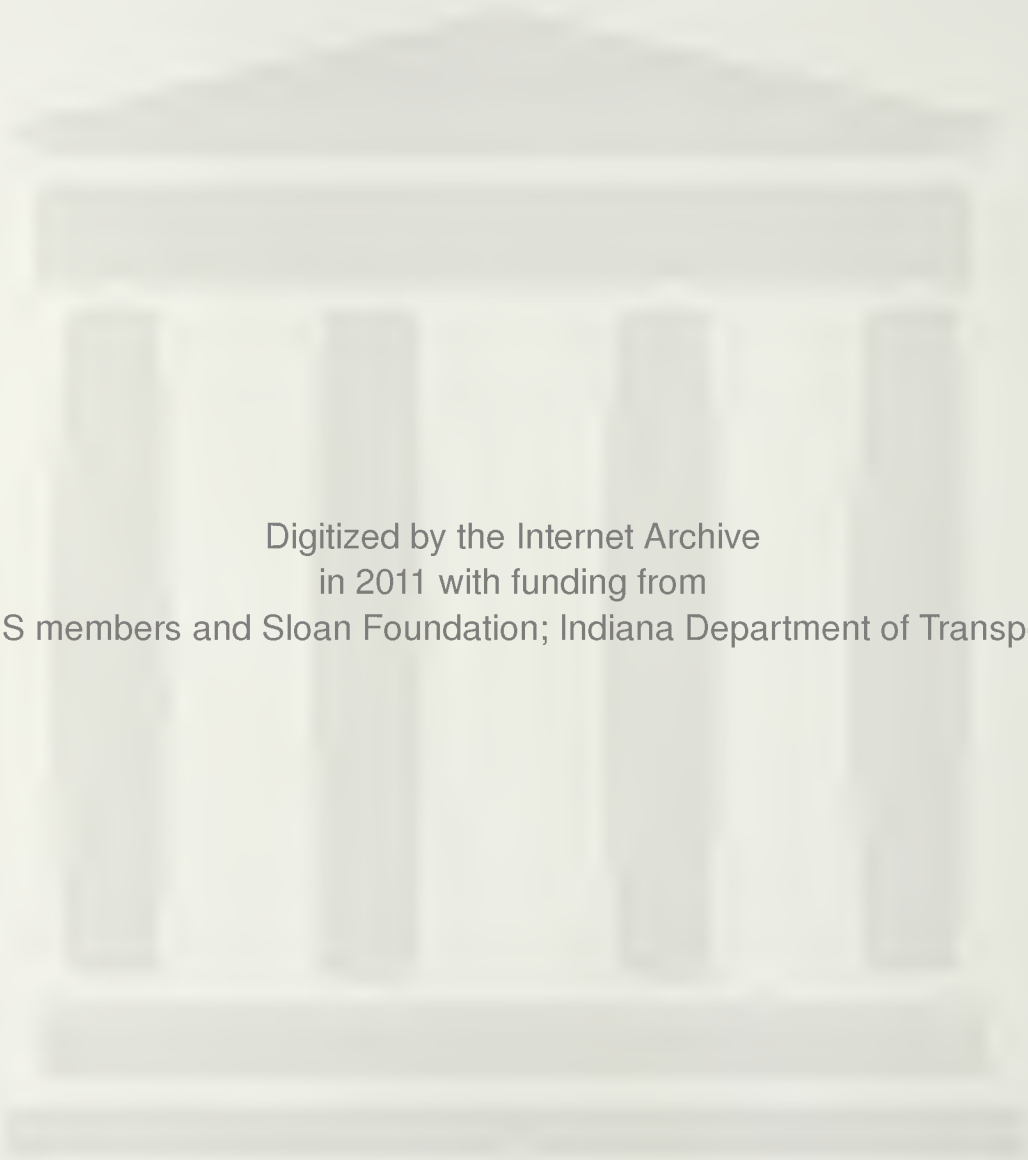

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Interim Report

"The Use of Fuzzy Sets Mathematics in
Pavement Evaluation and Management"

To: Harold L. Michael, Director
Joint Highway Research Project

August 23, 1984
Revised June 1985
Project: C-36-63J

From: A.G. Altschaeffl, P.E.
Research Engineer

File: 9-7-10

Please find attached an Interim Report entitled, "The Use of Fuzzy Sets Mathematics in Pavement Evaluation and Management." It was authored by Mr. M. Gunaratne, A.G. Altschaeffl, and J.L. Chameau of our staff. This is the first report on the HPR project entitled, "The Use of Fuzzy Sets Mathematics to Assist Pavement Evaluation and management."

This study proposes a methodology for ranking pavement sections according to maintenance urgency. Fuzzy sets mathematics is used to account for the human and system uncertainty inherently present throughout this process. Fuzzy sets are used to represent the subjectivity in pavement serviceability ratings and distress surveys, and the variability in Roadmeter, Skidtester and Dynaflect readings.

The attributes relevant to each category of maintenance are identified and an expert knowledge base containing priority values for known attribute value combinations is formed in collaboration with decision makers. A multi-attribute decision-making process is created to produce a crisp ranking of pavement sections according to maintenance urgency.

This report is presented for review and approval as evidence of partial fulfillment of the objectives of this project.

Respectfully submitted,

A.G. Altschaeffl, P.E.
Research Engineer

cc:
A.G. Altschaeffl
J.M. Bell
W.F. Chen
W.L. Dolch
R.L. Eskew
J.D. Fricker
M.K. Hunter
J.P. Isenbarger
J.F. McLaughlin
R.D. Miles
P.L. Owens
B.K. Partridge
G.T. Satterly
C.F. Scholer
K.C. Sinha
J.R. Skinner
C.A. Venable
L.E. Wood
T.D. White
S.R. Yoder

Interim Report

"The Use of Fuzzy Sets Mathematics in Pavement Evaluation and Management"

by M. Gunaratne
Graduate Instructor in Research
A.G. Altschaeffl
Professor of Civil Engineering
J.L. Chameau
Associate Professor of Civil Engineering

Joint Highway Research Project

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The contents of this report reflect the views of the authors who are responsible for the facts and accuracy of the data reported herein. The contents do no necessarily reflect the official views in policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

Purdue University
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16. Abstract

A methodology has been created for ranking pavement sections according to maintenance urgency using fuzzy sets mathematics to account for human and system uncertainty in the pavement management system. Fuzzy sets are used to represent the subjectivity in pavement serviceability ratings and distress surveys, and the variability in Roadmeter, Skid-tester and Dynaflect readings.

Techniques are presented for correlation of fuzzy numbers to create the Pavement Serviceability Index. Initial grouping of pavement sections compares the PSI with an Acceptable Serviceability Level and an Unacceptable Serviceability Level, both of which are subjective opinions obtained from experts.

The final ranking is formulated using fuzzy multi-attribute decision-making concepts using an expert knowledge base. This information is obtained from decision makers for known attribute value combinations using suitable questionnaires. The result is a unique ranking of pavement sections according to maintenance urgency.

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LIST OF SYMBOLS AND ABBREVIATIONS

AASHO - American Association of State Highway Officials

ADT - Average Daily Traffic

ANOVA - Analysis of variance

ASI - Acceptable serviceability index

ASR - Acceptable serviceability range

ASTM - American Standards of Testing Materials

BB - Benkelmen Beam deflection

CRC - Continuously reinforced concrete

DC - Direct current

DI - Distress index

D_i - Thickness of layer i

DMD - Dynaflect maximum deflection

DTN - Design Traffic Number

E - Elastic modulus

EAL_o - Initial equivalent axle load

ER - Engineering rating

ESAL - Equivalent single axle load

FI - Priority Index

FN - Friction number

G_i - Individual opinion of i

H - Degree of fitting

IDOH - Indiana Department of Highways

J - Fuzziness of a set

JRC - Jointly reinforced concrete

K - Kernel of fuzzification

K_o - Coefficient of lateral earth pressure at rest

L - Left side of a unimodal curve

M - Moderate damage

max - Maximum

min - Minimum

PCA - Portland Cement Association

PCR - Pavement condition rating

PMS - Pavement Management System

PSI - Pavement serviceability index

PSR - Pavement serviceability rating

p_t - PSI at time t

R - Right side of a unimodal curve

r_k - Relative rank for section k

R_{mk} - Maximizing priority set for section k

R_{pk} - Relative priority set for section k

RR - Roadmeter reading

Se - Severe damage

SI - Structural index

Sl - Slight damage

SN - Structural number

SPI - Index used for friction measured in spring

SUI - Index used for friction measured in summer

Sup - Supremum

T - Design thickness of a pavement

TRB - Transportation Research Board

u - Priority value

w_i - Weight of i

$w_{i_{18}}$ - Equivalent 18 kilo-pound axle load at time t

Y^h - h level set of Y

α - Concentration index

β - Half the support area of a symmetric fuzzy number

δ - Deflection

Δ - Fuzzified deflection

γ - Mean of a symmetric fuzzy number

ε - "Belongs to"

μ - Membership value

μ_A - Acceptability index

μ_N - Unacceptability index

Π - Product

σ - Standard deviation

θ - Distress rating

$\bigcup$ - Union of all elements in a discrete fuzzy set.

$\int$ - Union of all elements in a continuous fuzzy set

$\bigcap$ - Intersection of two sets

$\vee$ - Maximum of two memberships

$\hat{}$ - Minimum of two memberships

X - Cartesian product of two sets

∞ - Infinity

$\subset$ - Containment

EXECUTIVE SUMMARY

Introduction

This report presents the concepts and mathematical foundations for the use of new schemes with which to base decisions for pavement maintenance priorities. We believe that more information about the quality of pavement and the severity of the distress exhibited by the pavement is contained within these new schemes than in the conventional. This can allow the inclusion of more relevant information in the decision-making process than is now possible. This executive summary attempts to lay bare this process and show how the new schemes perform their function.

The many highway pavement sections in Indiana are in many different states of soundness or disrepair. Maintenance budgets, however, are limited. Thus, decisions must be made on which sections most deserve repair during a given year. In addition, criteria must be available for making these decisions. In Indiana, as well as in many other states, the objective of the maintenance program is to maximize pavement "rideability". The system essentially involves two steps: initial screening and decision making.

The initial screening is performed to identify sections requiring maintenance. This operation requires input from highway users and engineers on performance characteristics of pavement sections, and on the levels of those characteristics that suggest unsatisfactory performance. The screening process results in the characterization of pavement sections into several

maintenance categories.

The decision-making operation follows the initial screening to provide a rank-ordering of pavement sections within each maintenance category. The decision process requires the delineation of variables, criteria and attributes which are appropriate for each maintenance category. This information can only be provided by experienced engineers and decision-makers. Expert information is also needed to assess the interactions existing among the selected criteria and attributes. Once this expert knowledge base is established, it can be used by engineers to rank pavement sections for as long a time as the data are deemed relevant. Only performance and traffic data (i.e., values of attributes) are required, then, for the pavement sections to be examined.

In this report, techniques are proposed to acquire the knowledge base required by a pavement management system. Mathematical procedures are also developed to organize this information in a computerized decision-making model which makes allowance for the interactions among the different attributes.

This report has three goals:

1. To describe the mathematical techniques used in both the initial screening and the decision process;

2. To develop the framework (set of questionnaires) which can be used to acquire the expert knowledge base;

3. To provide simple numerical examples of application of the mathematical techniques (these examples are simple enough so that the reader can check them by hand calculations; they are provided to show that the mathematical intricacies are only basic algebraic operations). Note that all the mathematical techniques have been computerized for future use.

This report should be read in conjunction with the companion report by Andonyadis et al. (1985). The companion report describes how the mathematical techniques can be used in the pavement management system. It has four goals:

1. To provide simple physical interpretations of the mathematical techniques;

2. To use the answers to questionnaires presented in this report to acquire a typical knowledge base;

3. To show in selected examples how the knowledge base can be used to screen and prioritize pavement sections;

4. To make recommendations for future implementation of the proposed management system.

In this context, the following sections of this summary highlight the important steps of the methodology proposed in this report.

The reader who is not interested in the theoretical concepts behind these steps can concentrate on the companion report (Andonyadis et al., 1985) to see their use, making reference to this report as needed.

Initial Screening and User Input

Road users formally play a major role in evaluating the quality called rideability through the concept of the Pavement Serviceability Rating (PSR). The PSR reflects raters' opinions of the rideability of a selected number of pavement sections. Each rater is asked to state his view on the rideability of each section on a scale of 0 to 5 (poor to excellent); the PSR of the section is defined as the mean value of all raters' opinions. This subjective rating is the datum from which the maintenance program is developed, because everything that follows will tie back to it.

To reduce the need for many rating panels, a mechanical device, the PCA Roadmeter, that measures "roughness" is used on each rated section. A statistical correlation is then prepared between Roadmeter Reading and PSR and the rideability value that is predicted from the equation is called the Pavement Serviceability Index (PSI). Hence, all pavement sections can be screened efficiently by use of the Roadmeter. Then, the PSI of each section is compared to an Acceptable Serviceability Index (ASI) to determine the next course of action. In Indiana, the PSI is defined on a scale of 0 to 5, and 2.5 is used as ASI. Those

sections having PSI below 2.5 are considered excessively rough. This is the first decision point to sort out sections to be considered for maintenance in the existing framework.

Two observations deserve to be made at this point. First, the opinion of the rater contains uncertainty and imprecision, if only because judgment has vagueness attached to it in the quantitative sense. Secondly, different raters have different degrees of perceptiveness of what the roughness implies, e.g., someone who knows how pavements perform can infer that the roughness is caused by a defect that generally enlarges quickly and, thus, this is a hazard that requires quick attention. The entire gamut of road user deserves involvement in ratings, but advantage should be taken of the extra perceptiveness that some raters exhibit.

These thoughts can be included in the new scheme. The rater is not asked for a single value of rideability but for weights on a scale of 0 to 1 that he wants to attach to each possible rating value that is available to him. This represents his belief in each value and provides a central tendency to his opinion as well as a range to encompass the uncertainty in his judgment. Each rater can provide such a belief function, called the "membership function," for each section.

With expert information provided from the judgment of highway pavement managers, a perceptiveness weighting can be attached to each rater's opinions. The mathematical bases for assembling

all those various "opinions" are presented in this report. The result is a single, all-inclusive membership function for each pavement section. This will contain the spread caused by uncertainty as well as the effects of perceptiveness. Although the amount of information appears to look more complicated than that of existing techniques, so much more is contained in it, no one's opinion is discarded, and it can be computerized easily. If the ultimate judgment on rideability is that of the users, then, indeed, the "fuzzy sets" approach contains a full and thorough assembly of these judgments.

Let us turn, then, to the mechanical measurement of roughness. There is imprecision in the readings. This imprecision comes from both the random uncertainty in the measurement as well as from the human involvement in the procedures. This report addresses the correlation between Roadmeter Reading (RR) and the new "fuzzy PSR" in two ways: (1) as if RR were a crisp, deterministic, reproduceable number; and (2) as if RR were also a vague number described by a membership function to account for its irreproduceability and imprecision. Expert knowledge, through responses to questionnaires distributed to elicit the judgment of these experts, was used in the "fuzzification" of RR. The report provides a program to assemble RR data and to relate these data to the PSR data described earlier. This program allows the creation of the "fuzzy" PSI to describe each pavement section. At this point, then, each section is described as to roughness and rideability and these are related to the basic

rater opinions. The relation is a comprehensive one containing all the judgment about performance that can be extracted from the opinions.

The matter of what is an acceptable roughness, the ASI, is also one of judgment, and it represents the first decision point in the global decision process. Different people will recognize a section as hazardous (i.e., in need of maintenance) at different stages of roughness, as, for example, their perception of costs and degree of hazard differ. The new scheme makes allowance for this imprecision in the decision process. Experts were asked: (1) above what PSI value is a pavement totally acceptable for traffic; (2) below what PSI is a pavement totally inadequate. The responses were used to, first, create an Acceptable Serviceability Range. This Range contains the varied judgments of the different experts as to what is acceptable. Similarly, a Non-Acceptable Serviceability Range is also created; this one is not necessarily the complement of the other, because judgments are involved and there is a domain of PSI values where decisions on acceptability are difficult to make. These two ranges are membership functions which contain a complete representation of the judgment and experience of the experts.

This report contains the mathematical bases for comparing the fuzzy PSI of a section with the fuzzy Acceptable Serviceability Range. An index describing how well the section "belongs" to the acceptable range is obtained for each section. Also obtained is a separate index for each section describing how well it

belongs to the unacceptable range. The criterion in the report says if the acceptability index is the larger, the section has acceptable roughness.

The skid resistance of pavements with acceptable roughness is measured to obtain a friction number used to identify sections which are too smooth for safety. Four sources of variation affect the measurements made with a skid-tester: (1) inability of repetition; (2) variations along pavement sections; (3) uncertainty associated with conversion favors; and (4) variability due to statistically insignificant factors. It is shown in this report that, although part of this uncertainty is random in nature, system uncertainly also plays a major role and several procedures are suggested to make allowance for it. Following the approach already taken for RR and PSI, expert opinions were sought again on what is acceptable and unacceptable FN. This is followed by the assembly of those sections requiring attention, in accordance with the previous decision criterion.

For the initial screening of pavements, the "fuzzy performance data", fuzzy PSI and FN, are compared with acceptable and unacceptable serviceability and friction ranges, respectively. The comparison technique provides indices describing the degree of belongingness of a given pavement section to the acceptable and unacceptable ranges, respectively.

This provides a criterion to classify pavement sections into three categories:

	1st	2nd	3rd
PSI	OK	No	OK
FN	No	N/A	OK

Today, in Indiana, category II pavements are ranked using the PSI-RR data, FN, and traffic count (ADT). This report recommends inclusion of distress surveys. It shows how to create the membership functions which contain the judgment of each survey member on various aspects and types of distress. These components are weighted, and the results of crew members are assembled for each section. We, thus, have a fuzzy Pavement Condition Rating (PCR); it contains the combined judgments of all crew members, including their individual different perceptiveness on the import of the distress, and is a description of the distress exhibited by the section. The procedures have been created to allow inclusion of distress severity in the maintenance ranking procedure.

The Decision Process

At this stage, the goal of a pavement management system is to provide decision-makers with a ranking of pavement sections in any desired category. The ranking, or state, of a pavement belonging to any one of the three categories can be represented by a number of attributes that the decision-makers believe to be important for a decision on maintenance urgency.

For category I sections, FN, average daily traffic (ADT), and approximate cost have been deemed decision variables (or attributes). Assuming cost is related to FN, the two main attributes are FN and ADT. For category II sections, the report presumed PCR, ADT, and cost. Assuming that the cost is a function of the PCR and deflection measured under the Dynaflect, the three main attributes for this category are PCR, ADT, and deflection. For category III pavements, future service life is the key issue. Using presently established performance vs. time curves (PSI or FN vs. time), the service lives of each section can be assessed as a fuzzy number because of the imprecise nature of the input variables. These two attributes can serve in the decision process, and a ranking can be made on the basis of perceived need for future maintenance.

The selection of attributes in this study for each category was guided by present practice in IDOH. The proposed technique is not limited to these attributes. If it is felt desirable, the Indiana Department of Highways may remove some of these attributes or add other attributes. This only requires the development of the knowledge base for the new attributes, following the same approach used in this report for the above attributes. This is further discussed in the report by Andonyadis et al. (1985) where ADT, FN and PCR are used for the first category of pavements, and ADT, PSI and PCR for the second category. These latter selections were guided by discussion with engineers from IDOH and by the responses to the questionnaires.

The key to the decision making scheme presented in this report is the creation of the component of the knowledge base that can be labelled "utility functions." Techniques have been developed to construct this knowledge base from the responses of highway experts to questions such as: "If the PCR is 70.0 and the dynaflect reading is 0.001 inch for an unacceptably rough pavement with an ADT of 3000, what relative priority would you assign on a scale of 1-10?". An expert can assign such a subjective value based on heuristic rules that have come through years of pavement management experience.

A matrix of decision criteria is created from the decision-makers' judgment of relative priorities obtained for a selected combination of attribute values relevant to each category. Then, using the techniques presented in the report, the assembly of attribute data is related to the expert knowledge base to rank pavement sections within any of the three pavement categories.

It is important to note that the ranking provided by the proposed decision-making scheme is crisp. For example, processing of PCR, ADT and deflection data available for 50 sections within the second category will result in a ranking of these sections from 1 to 50. The section with the lowest rank requires maintenance first.

Concluding Remarks

The new scheme proposed in this report is founded upon the judgments of experts in various aspects of pavement performance.

There is much uncertainty present yet today in the understanding of this performance, i.e., judgment is, indeed, involved in establishing maintenance urgency. The fuzzy sets mathematics appears to be very effective in handling this uncertainty and judgment. It is fully consistent with the manner in which decisions are made, and it creates a crisp ordering of pavement sections according to maintenance priorities. Because more information about quality of pavement and severity of distress is contained in this scheme than in the conventional, the authors consider this scheme a major improvement and worthy of implementation.

The knowledge base required by the new scheme is composed of five parts:

1. variability in PSR, RR, FN, PCR;
2. ratings of a panel of users;
3. PSR-RR relationship;
4. acceptable and nonacceptable levels of PSI and FN;
5. utility values (i.e., the matrix of decision criteria).

It is important to note that once the knowledge base is established, the performance and traffic parameters for the pavements to be ranked are the only data required in the analysis. This is illustrated in Figure E.1, which shows a flow-chart of operation. The user's intervention is limited to the left side of the flow

chart (input). The knowledge base and mathematical operations are entirely computerized.

As in all decision-making, the knowledge base and criteria do deserve re-examination periodically. A given knowledge base is available for use as long as IDOH considers the contents to be relevant to pavement management. It can be changed readily when new data appear more appropriate or if the state-of-the-art and/or experts' judgement changes. This would require only the development of the related component of the knowledge base, using the same approach as herein. Following this, ranking procedures are the same, using the improved reference datum.

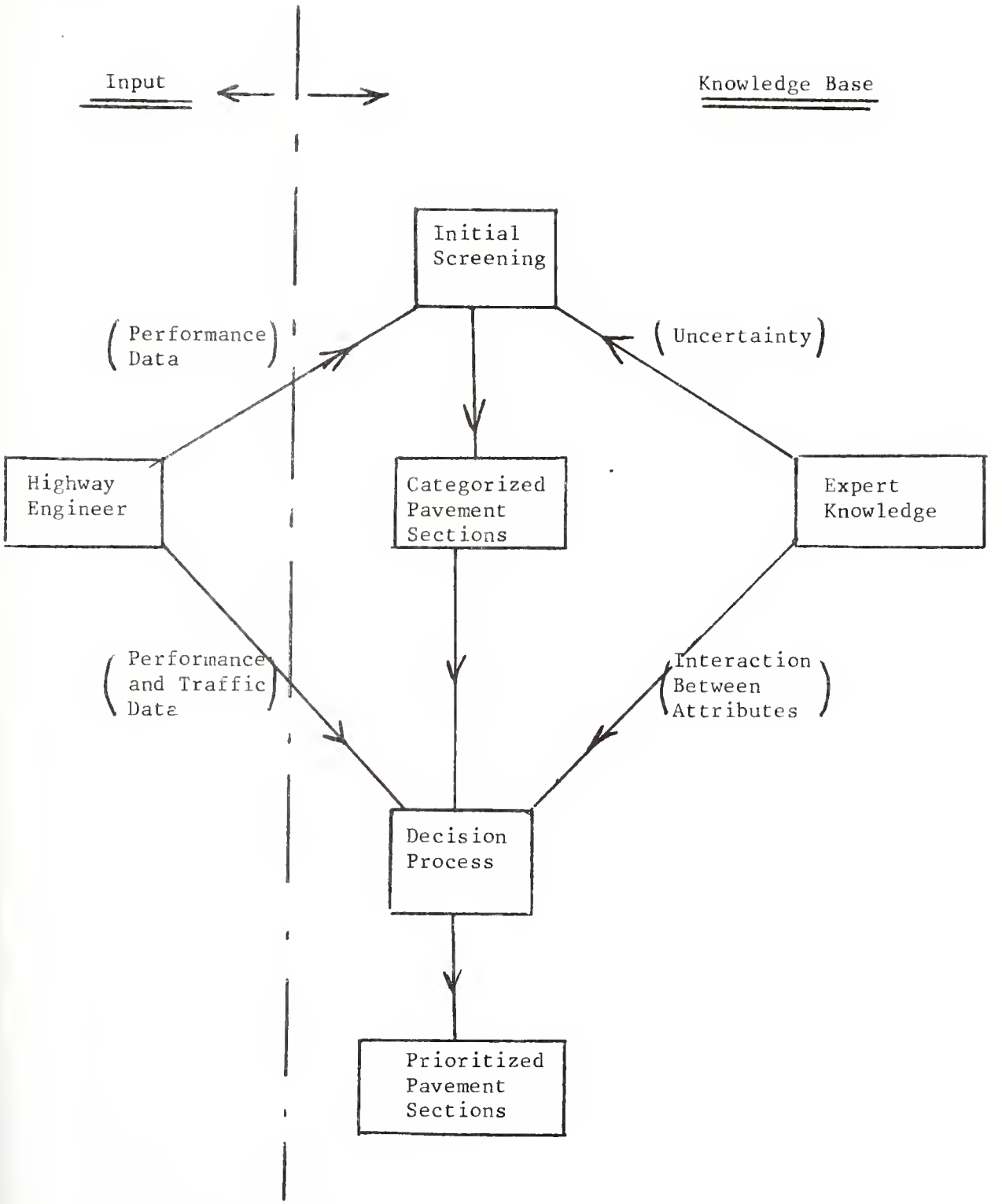


Figure E.1: Decision Flow Chart.

CHAPTER 1 : INTRODUCTION

Uncertainty in engineering

The practice of engineering has required the use of experience and judgment throughout history. As scientific knowledge increased, engineering gained a more scientific base, but rules of thumb and engineering judgment are still used extensively. At present, theoretical or empirical techniques are available to assess an engineering problem and make technical decisions. However, even the most sophisticated techniques cannot take into account all the issues of the problem. This is the case especially with uncertainty that may be involved in certain components or variables. Consequently, engineering decisions are often based on a combination of objective scientific knowledge and subjective engineering judgment regarding uncertain areas.

The uncertainty surrounding any engineering problem is basically of two types. One is random uncertainty , which derives entirely from the random nature of parameters describing a system. Analytical techniques embodied in reliability theory concentrate on random uncertainty. The other type, which covers human based uncertainty and system uncertainty derive from a "vagueness" of a proposition, or a lack of precision of an event or a lack of understanding of a system. To analyse this type of

uncertainty, a mathematics which is directed at "vagueness" is required. This is the potential role of fuzzy sets theory. Engineering activities such as pavement management, which exhibits these characteristics of uncertainty, can certainly benefit from the use of this theory.

Pavement mangement system

In view of the major national investment in the highway network, a sound pavement management system (PMS) is a major concern of civil engineers in the country. It is a tool that provides decision-makers at all management levels with information and procedures necessary to optimize the design and maintenance of pavements at the network level. Therefore, evaluation of project pavement performance, priority establishment and selection of maintenance and rehabilitation strategies, all fall within the framework of a pavement management system. In other words, a PMS should permit determination of the road on which an action is needed, the type of action required, and when this action should be scheduled.

An overview of various pavement management systems reveals that a PMS is highly specific and particularly structured to the attitudes, needs and procedures of the implementing agency. Consequently, many of the details of a PMS must be fitted to or molded by the implementing agency itself. Nevertheless, significant portions of the development and implementation work involved in organizing and operating a PMS are potentially applicable to a

wide variety of uses. Most highway agencies collect the following performance data which are primarily used to assist in making decisions on pavement management and rehabilitation:

1. Roughness or rideability

2. Skid resistance

3. Structural adequacy or deflection

4. Surface distress manifestation.

The Arizona Department of Transportation uses a Mays-meter, a Mu-meter and a Dynaflect to moniter roughness, skid resistance and deflection, respectively. Distress surveys are conducted visually by a crew using a crack guide. In Arizona, they use all four kinds of data along with age and average daily traffic counts (ADT) of the pavement, to obtain a score for maintenance priorities.

In California, PCA meters and skid-testers are used for making rideability and skid measurements. Distress is determined by a crew, but deflection measurements are not performed on a systematic basis. Priority categories are formed by considering ride score, distress data and ADT.

Mays-meter, Dynaflect and a towed two-wheeled trailer conforming to ASTM E 274-40 are used by the Florida Department of Transportation to measure roughness, deflection and skid resistance. The observation of rutting, cracking and patching by a

crew of raters results in distress surveys. An engineering rating (ER) is determined from the above measurements, which along with the cost effectiveness (determined by ADT, length of pavement and project costs) determines priorities for maintenance.

The New York Department of Transportation does not have a priority assignment phase in its PMS. Maintenance action is based on a pavement serviceability system where rideability data are obtained by a sophisticated DC differential transducer mounted on the floor of a wagon. Skid data are collected only to corroborate the need for correction of excessively slippery pavements, i.e. where the frequency of accidents is high. Distress and deflection data are not currently collected in the state.

The testing sequence of the pavement management system of Indiana is shown in Figure 1. Here, as in most other states, pavements are evaluated on the basis of all four properties. A PCA roadmeter is used to measure the roughness which is transformed into a serviceability scale; serviceability criteria are used for the initial screening of road sections. Friction testing is then conducted on pavements with acceptable roughness, whereas unacceptably rough pavements are surveyed for distress and deflection. Pavements without adequate skid resistance are immediately placed in the group of pavements needing prompt attention. Priorities for the present year would next be established using the results of these different evaluations. For those pavements not requiring immediate attention it is neccessary to estimate remaining friction and serviceability lives.

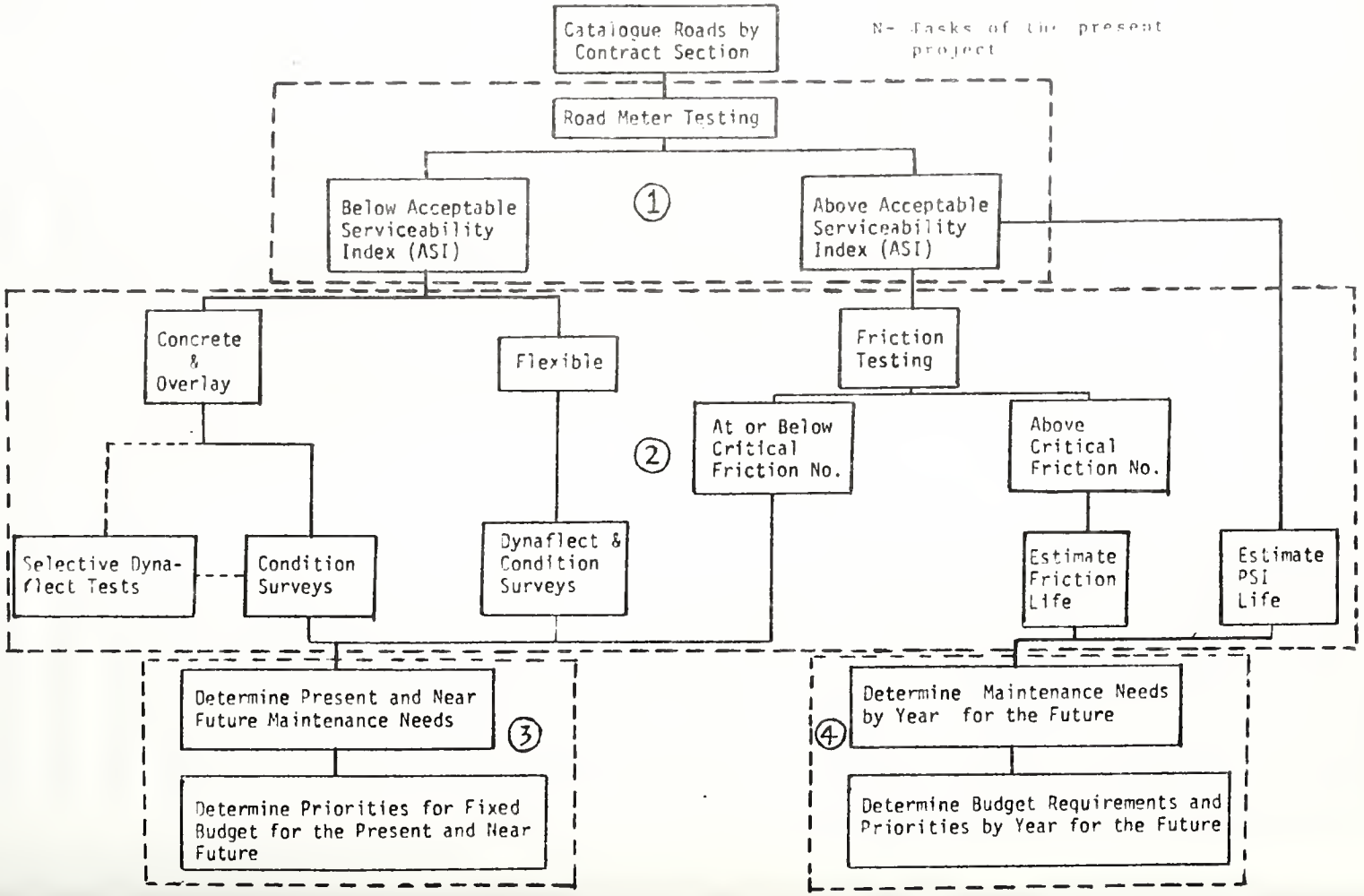

Catalogue Roads by Contract Section
N- Tasks of the present project
Road Meter Testing
Below Acceptable Serviceability Index (ASI)
1
Above Acceptable Serviceability Index (ASI)
Concrete & Overlay
Flexible
Friction Testing
2
At or Below Critical Friction No.
Above Critical Friction No.
Selective Dyna-flect Tests
Condition Surveys
Dynaflect & Condition Surveys
Estimate Friction Life
Estimate PSI Life
Determine Present and Near Future Maintenance Needs
3
Determine Priorities for Fixed Budget for the Present and Near Future
4
Determine Maintenance Needs by Year for the Future
Determine Budget Requirements and Priorities by Year for the Future

Figure 1. Testing Sequence of Indiana's PMS

Then, priorities for future maintenance can be assigned.

Purpose and scope

Presently, statistical models are used to predict the effects of various environmental factors on pavement properties and to correlate objective data in the evaluation process. Stochastic approaches to pavement management problems suffer from shortcomings, because of the following inherent characteristics of these problems :

1. These problems involve the complex socio-economic environment ;
2. All the variables are not completely defined or cannot be precisely measured ;
3. Personnel bias and subjectiveness enter in the analysis ;
4. Criteria for performance are often defined in a vague manner.

It is the author's belief that fuzzy sets can be incorporated in the PMS to supplement present analytical models. This modification has the potential of meeting the demand for a more effective and a methodical PMS, which is essential in view of limited budget allocations for highway maintenance.

The purpose of this investigation is to provide support and procedures for the use of fuzzy sets in the areas where human and

system uncertainty exist in Indiana's pavement management system. The mathematical techniques developed in the process are meant to be applied to other civil engineering problems, and even other engineering fields as well.

Areas where fuzzy sets can play a major role in Indiana's PMS are marked by dashed lines in Figure 1. The scope of this research project covers the following tasks :

1. Formulation of a fuzzy PSI model and the initial screening of sections based on rideability ;

2. Introduction of fuzzy sets concepts to account for variablity of measuring instruments ;

3. Identification of pavements needing prompt attention and use of fuzzy decision making techniques to allocate maintenance priorities ;

4. Introduction of fuzzy sets concepts in decision making on future priorities.

This report discusses these tasks and presents the development of appropriate procedures to account for uncertainty in the system. It is hoped that the PMS will be enhanced by such procedures for the long term economic benefit of Indiana.

CHAPTER 2 : FUZZY SETS

Part of an engineer's technical capability has to be the ability to choose appropriate theoretical models for representing physical systems. This is known as system identification. Easily modelled components of a system can be analysed by known engineering techniques. However, the closer one looks at a system, or the larger the system is, the more complex it becomes. In both cases, components of the system or variables within the system become imprecisely defined, i.e vague. Experience can play a vital role here, but mathematical tools are needed to incorporate the information it provides, in the analysis of the system. Experience often results in statements such as "highly variable" and "more or less safe" which present inherent vagueness or "fuzziness". The mathematics of fuzzy sets was developed to treat "fuzzy" statements and the fuzziness inherent in large or complex systems.

A fuzzy set is a class of objects where the objects do not have a well defined criterion of belongingness (membership) to that class. "The class of tall men" or in a civil engineering sense "a collection of sites suitable for a building", are examples of fuzzy sets.

The fuzzy sets theory, introduced in 1965 by Zadeh, is intended to provide a systematic procedure to handle classes of

information in which the transition from membership to nonmembership is gradual rather than abrupt.

Membership function

If S is a conventional set in the space X, the usual characteristic function of S is defined as follows :

$$f(x) = 1 \quad \text{if} \quad x \in S \tag{1a}$$

$$f(x) = 0 \quad \text{if} \quad x \notin S \tag{1b}$$

where the notation $\in$ means "belongs to". Equation (1) corresponds to a yes-no decision as to whether a given value x is a member of a given set S. On the other hand, if A is a fuzzy set in the space X, the characteristic function becomes a membership function $\mu_A(x)$ which associates with each point in X, a real number in the interval [0,1]. The value of $\mu_A(x)$ at x represents the "grade of membership" of x in A; the nearer the value of $\mu_A(x)$ is to unity, the higher is the grade of membership of x in A. If it takes a value of unity for any x, then A is a normalized fuzzy set. In a mathematical sense, $\mu_A(x)$ is a "restriction" on the set of possible values that the variable x can take (Blockley et al.,1983). When A is a well defined (or ordinary) set, $\mu_A(x)$ can only take the two values 1 or 0 according to whether x does or does not belong to A.

The grade of membership $\mu_A(x)$ can also be considered as the degree of support or belief that the element x belongs to the subset A. As an example, let X be all levels of design quality

which may range from poor (x = 0) to excellent (x = 10) (Brown and Yao, 1983). Strong and weak support (belief) for a particular structure to be well designed can be expressed by the subsets A and B, respectively, as shown in Table 1.

A more convenient notation than a table is usually adopted to represent fuzzy sets:

$$A = \underset{j}{U}\ \mu_A(x_j)/x_j \tag{2a}$$

$$= \mu_A(x_1)/x_1 + \ldots + \mu_A(x_n)/x_n \tag{2b}$$

where the + sign is used in place of union as in ordinary set theory, and $\mu_A(x_j)$ is the degree of support for the value x_j. As an example, the fuzzy set B for weak support in Table 1 can be written as:

$$B = 0.1/3 + 0.4/2 + 0.7/1 + 1./0$$

For continuous fuzzy sets, equation (2a) becomes :

$$A = \int \mu_A(x)/x \tag{3a}$$

In the conventional set theory, sets as well as operations on sets can be illustrated on Venn diagrams. Fuzzy sets are conventionally depicted by a plot of membership function against the variable. Figure 2 shows such a plot for a continuous fuzzy set A in the space X.

Table 1. Supports for a 'good' design

Design quality X	10	9	8	7	6	5	4	3	2	1	0
strong support A	1	0.7	0.4	0.1	0	0	0	0	0	0	0
weak support B	0	0	0	0	0	0	0	0.1	0.4	0.7	1

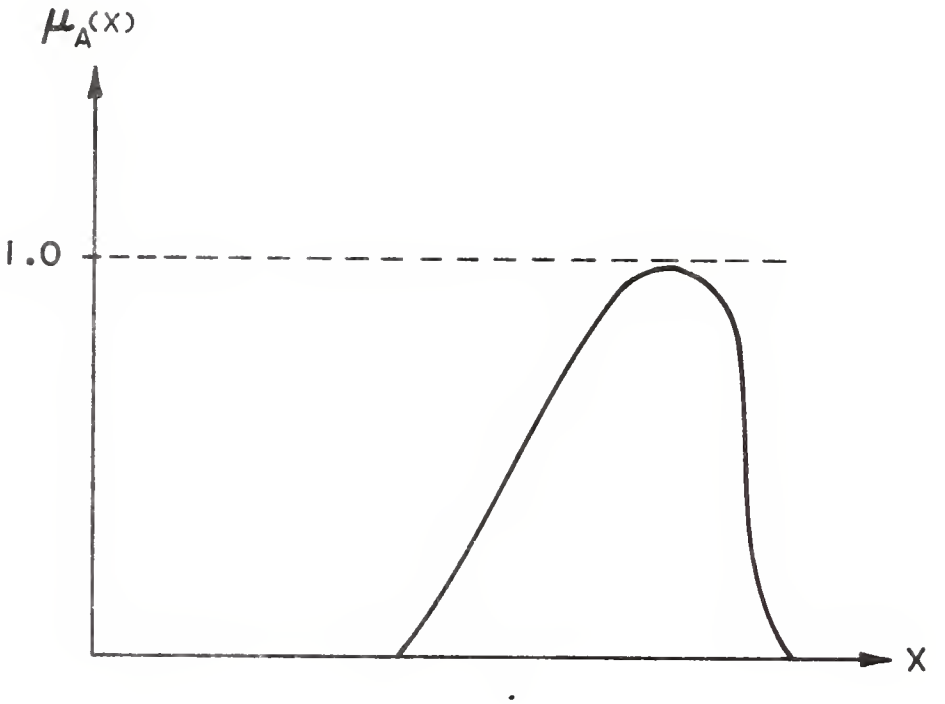


Figure 2. Membership function

The problem of estimating the membership functions for actual problems has not been systematically studied in the literature. Although many ideas and methods have been suggested by several authors independently (Dubois and Prade,1980), there is as yet no widely agreed upon rule, to estimate memberships. From the methods presently available, two techniques, namely exemplification (Zadeh,1972), and implicit analytical method (Kochen and Badre, 1976) were found appropriate and are extensively used in the present work. The basic concept of the former will be briefly described below and the latter will be discussed in Chapter 3.

Assume that X is an ill-defined set and x_i (i=1,n) are the possible events. In the method of exemplification, a number of experts are asked whether "x_i is X". There will be different linguistic responses such as "true", "more or less true", etc., for which values such as 1.0, 0.75 are pre-assigned. Then the average of these values is considered as $\mu_X(x_i)$. This procedure can be repeated for all x_i values to obtain the membership function.

Saaty (1977) developed a pairwise comparison method, where the degrees of support of two events (in an ill-defined set) are compared at a time. Then, a comparison matrix, [A] (nxn) is constructed where the element a_{ij} is the ratio of the degrees of support for x_i and x_j, in X. If $\underset{\sim}{W} = (w_1,.....,w_n)$ is the vector containing the n-wise comparisons or the relative weights (w_i) of each x_i in comparison to all x_i, Saaty showed that the maximum

eigenvalue (λ_{max}) in the following expression :

$$[A - \lambda I]\ \underset{\sim}{W} = 0 \qquad (3b)$$

produces the required w_i values. Then the memberships $\mu_x(x_i)$ can be obtained by stretching w_i values on a zero-unity scale.

Fuzziness and probability

One may wonder whether such a graphical representation (Figure 2) of a membership function is similar to a probability density function of a random variable. A probability measure p(x) describes the uncertainty in occurence or randomness of an event x, well defined in the space X; a membership function $\mu_A(x)$ provides a criterion for the belongingness of any value in the space X, in an ill-defined set A. These two measures originate from very different concepts. Furthermore, since all mutually exclusive events in a space are exhaustive, p(x) values for all x in the space X would sum to 1. No such restriction can be imposed on $\mu_A(x)$, due to the fuzziness in the set A. In fact, Zadeh (1978) introduced the ideas of a theory of possibility which is distinct from probability theory, based on fuzzy set theoretical concepts. A membership function $\mu_A(x)$ imposes a restriction on the possibility of a set A taking a value x. Therefore, an expression of a possibility distribution can be viewed as a fuzzy set and such a distribution may be manipulated by rules of fuzzy sets.

Fuzzy sets theory has been attacked by some who proposed that it is probability in disguise. Oden (1977), however has

shown that this is not the case. The finding of his psychological studies indicate that these two concepts are handled very much differently in the human mind. For example, the idea that a man whose height is 6 feet has a membership of 0.9 in the fuzzy class of "tall men", should not be confused with the idea that 90% of men who are 6 feet tall are "tall men", while the other 10% are not.

Connectives

Many complex problems can be divided into a sequence of simpler questions which can be answered by experienced individuals using their subjective judgment. These responses can then be converted to membership functions and manipulated following the theory of fuzzy sets to obtain a meaningful answer to the originally complex problem. In the following, basic manipulation techniques are presented, and illustrative examples for some of them are found in Brown et al(1983), and Yao et al(1981).

Two basic options are available for aggregating fuzzy sets. For the case of two fuzzy sets A and B, the "optimistic" aggregate (union) assumes credibility in opinions expressed in either A or B, while the "pessimistic" aggregate (intersection) assumes credibility only in the combined opinion of A and B. Union and intersection are denoted as :

$$C = A \cup B \qquad (4a)$$

$$D = A \cap B \tag{4b}$$

Shaded areas in Figures 3a and 3b show the possible regions of membership for C and D respectively, following their definitions. If A and B have membership functions of $\mu_A(x)$ and $\mu_B(x)$ respectively :

$$\mu_C(x) \geq \max (\mu_A(x) , \mu_B(x)) \tag{5a}$$

$$\mu_D(x) \leq \min (\mu_A(x) , \mu_B(x)) \tag{5b}$$

Fung and Fu (1975) justify the choice of lower bound in equation (5a) for union and upper bound in equation (5b) for intersection, using an axiomatic approach. Thus :

$$\mu_{AUB}(x) = \max (\mu_A(x) , \mu_B(x)) \tag{6a}$$

$$\mu_{A.B}(x) = \min (\mu_A(x) , \mu_B(x)) \tag{6b}$$

It is common to indicate "max" and "min" as v and ^ respectively, and the above equations can be rewritten as :

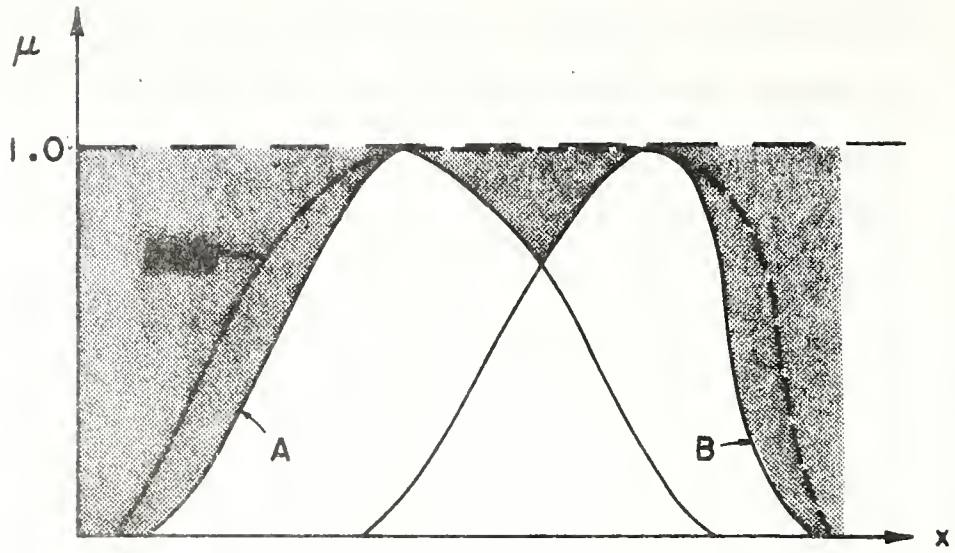
$$\mu_{AUB}(x) = \mu_A(x) \vee \mu_B(x) \tag{7a}$$

$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x) \tag{7b}$$

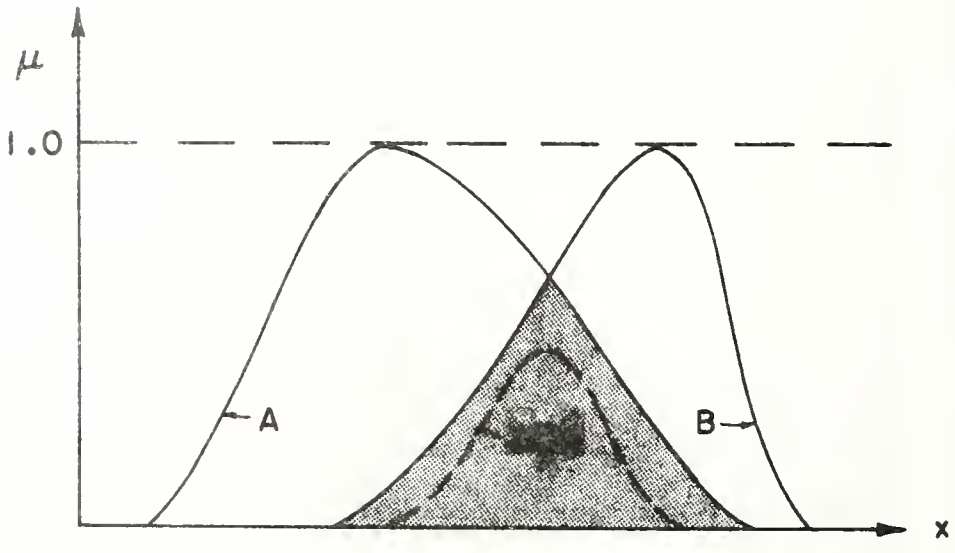
Differences in the basic concepts of probability and possibility becomes evident when comparing this equation to the corresponding axioms for probability. If A and B are mutually exclusive sets :

$$P(A \cup B) = P(A) + P(B) \tag{8a}$$

If A and B are statistically independent :



a) A U B



b) A ∩ B

Figure 3. Possible memberships for union and intersection

$$P(A \cap B) = P(A).P(B) \tag{8b}$$

Interactive operators that reflect a trade-off between A and B, as opposed to "min" and "max" have also been defined. Two frequently used operators are the algebraic product and the probabilistic sum operations defined below:

$$\mu_{A.B}(x) = \mu_A(x).\ \mu_B(x) \tag{9a}$$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\ \mu_B(x) \tag{9b}$$

In a real problem, the analyst might prefer the interactive aggregation types, as the individual memberships directly influence that of the aggregation. However, they suffer from the drawback that probabilistic and fuzzy reasoning are both combined therein. On the other hand, union and intersection operations involve fuzzy logic only, though they are non-interactive. Dubois and Prade (1978) state that, strictly, a fuzzy model does not take into account the number of ways in which an event can occur, only the fact that the occurence may happen. Thus, min-max operations are more logical for fuzzy set theoretic work than interactive operators.

Fuzzy numbers

In most practical problems there are invariably some parameters or variables which are imprecisely defined. This is usually the case when subjective and objective tolerances are involved in the assigned values of some components of a system. Examples of such cases in civil engineering practice are limiting the

distortion to about 1 in 75 in settlement analysis, using a coefficient of earth pressure at rest (K_o) of about 0.5 for stability calculations of embankments, etc. Certainly the engineer would like to see the effect of uncertainty surrounding these values on the final answer, i.e safety of the building in the first case and safety of the embankment in the latter case. This can be achieved by representing such fuzzy subjective values, or fuzzy numerical data from imprecise measurements, by means of fuzzy numbers.

Mathematically, a fuzzy number is a fuzzy subset of the real line whose highest membership values are clustered around a given real number called the mean value. The membership function is monotonic on both sides of this mean value, spreading throughout a tolerance interval determined through expertise on the problem area or statistical data. This approach is of immense practical interest since algebraic operations on fuzzy numbers can be easily performed using the extension principle, once the membership functions are determined.

Dubois and Prade (1978) describe the general shape of the membership function μ_n for a fuzzy number $\tilde{n}$ on the real line R as follows. μ_n is :

i. a continuous mapping from the real line R to [0,1]

ii. zero for all x in $]-\infty,c]$

iii. strictly increasing in [c,a]

iv. constant in [a,b]

v. strictly decreasing in [b,d]

vi. zero for all x values in $[d, \infty[$

This is graphically shown in Figure 4a. $\mu_n(x)$ can be interpreted as the truth value of the assertion "the value of $\tilde{n}$ is x". Two special cases are of practical interest,

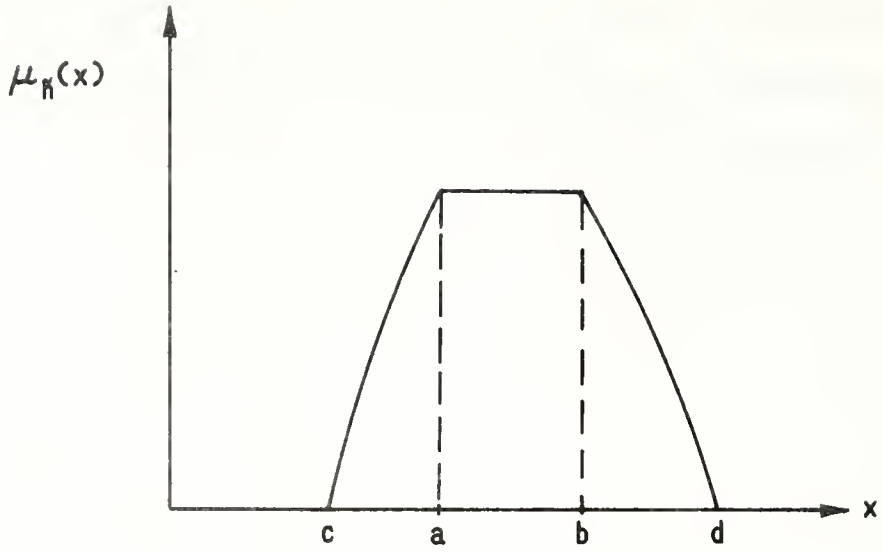
1. If a=c=b=d , $\tilde{n}$ is an ordinary number.

2. If a=b , $\tilde{n}$ represents a value "approximately a" and is shown in Figure 4b.

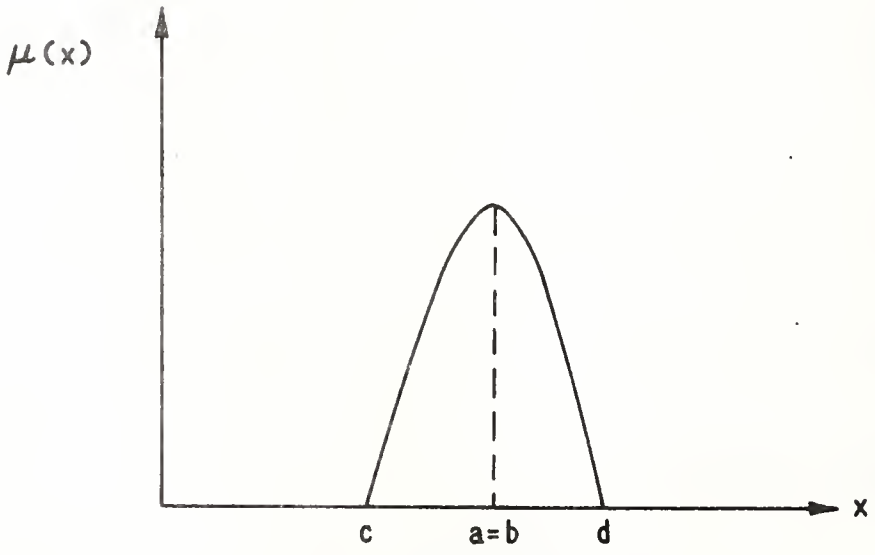
The latter type is known as L-R (left-right) type fuzzy numbers, since two algebraic functions of L and R can be employed to mathematically define the left and right halves of $\mu_n(x)$. In the absence of additional information, it is logical to represent memberships of "approximately a" by a symmetric function where a=b=1/2 (c+d). A wide variety of algebraic functions can be selected to define such an L-L or an R-R function. In the work presented herein, L-L (or R-R) fuzzy numbers are used extensively, and parabolic curves known as π curves (Zadeh, 1975), are selected for reasons described in Chapter 3.

Fuzzy relations

Very often the relationship between two mutually dependent classes or variables is neither very exact nor very inexact. In other words, the linkage between objects in the two classes, or values taken by the two variables, varies gradually from a condition of very weak to very strong. Such situations can arise



a) Fuzzy number $\tilde{n}$



b) Representation of 'approximately a'

Figure 4. Fuzzy numbers

basically from two sources : (1) fuzziness involved in the related classes or variables, and (2) approximate reasoning used to define the relationship.

An example for a fuzzy relation originating from the latter source is a statement such as "if the stiffnesses of the two layers are approximately equal, danger from crack development is minimal", often made by engineers with regard to the safety of dams from cracking.

If X and Y are two such "fuzzily related" variables, the linkage between a value x of X and a value y of Y can be expressed by a number in the interval [0,1]. This imposes a restriction on R with respect to any pair (x,y) in a mathematical sense and thus is defined as the membership characterizing the relationship. If the relationship involves only two variables, obviously the membership function is bivariate and the relation is a binary one in the space X x Y. According to the previous notation this is expressed as :

$$R = U\ \mu_R(x, y)\ /(x, y) \tag{10}$$

for the discrete case, and

$$R = \int \mu_R(x, y)\ /(x, y) \tag{11}$$

for the continuous case.

Since a graphical representation corresponding to Figure 2 would be three dimensional, a matrix form is a more convenient representation. For example, the previous statement regarding

cracking can be depicted by the matrix shown in Table 2. Here, the stiffness of a clay layer is assumed to be indicated by its elastic modulus (E_i).

More generally if $X_1,.....,X_n$ are n universes, an n-ary fuzzy relation R in X_1 x.....x X_n is a fuzzy set on X_1 x....x X_n denoted as :

$$R = U\ \mu_R(x_1,..,x_n)\ /(x_1,....,x_n) \tag{12}$$

where $\mu_R(x_1,...,x_n)$ is a multi-variate membership function.

Let us now consider the method of forming a fuzzy relationship from two given mutually related fuzzy sets. Let A and B be fuzzy sets in the spaces of X and Y respectively, and x and y be any possible values of A and B. Consider the following propositions :

$$x \text{ belongs in A with a restriction } \mu_A(x) \tag{13a}$$

$$y \text{ belongs in B with a restriction } \mu_B(y) \tag{13b}$$

According to Zadeh (1975), the rule of implied conjunction asserts that, in the absense of additional information concerning these propositions, equations (13a) and (13b) taken together implies that :

$$x \text{ belongs in A and } y \text{ belongs in B with a restriction } \mu \tag{14}$$

Also, in the absense of additional information, the rule of maximal restriction asserts that equation (14) implies that :

$$(x,y) \text{ belongs in AxB with a restriction } \mu \tag{15}$$

Table 2. 'Approximately equal stiffnesses'

E_2 (kg/cm^2) \ E_1 (kg/cm^2)	40	45	50	55
40	1.0	0.6	0	0
45	0.6	1.0	0.6	0.2
50	0	0.6	1.0	0.6
55	0	0.2	0.6	1.0

with the value of μ given by the expression :

$$\mu = \min [\mu_A(x) , \mu_B(y)] \tag{16}$$

since "min" is characteristic of the "and" connective under maximal restriction. A x B is defined as the cartesian product of A and B, and from equation (16) it follows that :

$$\mu_{AxB}(x,y) = \min [\mu_A(x) , \mu_B(y)] \tag{17}$$

Therefore, when additional information is not available this provides a convenient way of forming a relationship between A and B, with the membership function given by equation (17). Moreover, if A and B belong to the same space, the implication in equation (15) would reduce to :

$$x \text{ belongs in } A \cap B \text{ with a restriction } \mu \tag{18}$$

due to the implied "and" connective in equation (14).Then, membership of $A \cap B$ is consistent with that given by equation (16).

Extension principle

The extension principle introduced by Zadeh (1975) is one of the most basic ideas of fuzzy sets theory. It provides a general method for extending non-fuzzy mathematical concepts to deal with fuzzy quantities. The fundamental difference between classical (precise) set theories and fuzzy sets theory is that, in a precise theory a variable has a value whereas in the latter a variable has a degree of membership attached to each possible value.

It follows that a fuzzy variable is completely defined by its membership function. This is clearly expressed in the extension principle. By applying this principle, algebraic operations for fuzzy numbers can be developed conveniently.

If f is a mapping from X_1 x.....x X_r to a universe Y such that $y = f(x_1,.....,x_r)$, the fuzzy sets $A_1,.....,A_r$ in respective spaces $X_1,.....,X_r$, will induce a fuzzy set B on Y through f. The extension principle allows us to determine the membership function for B in terms of those of A_i, by the following expression :

$$\mu_B(y) = \sup_{y=f(x_1,..,x_r)} \min [\mu_{A_1}(x_1),.....,\mu_{A_r}(x_r)] \qquad (19)$$

For the unary case, where f is a mapping from X to Y such that y=f(x), equation (19) reduces to :

$$\mu_B(f(x)) = \mu_A(x) \qquad (20)$$

which will be frequently used in this work.

This principle can also be applied to a mapping of "dependent" fuzzy variables, i.e. when a specified relationship exists between the variables. For simplicity, the case of two dependent variables is presented here. If A_1 and A_2 are mutually dependent fuzzy sets in X_1 and X_2, which induce a fuzzy set B in Y, then:

$$\mu_B(y) = \sup_{y=f(x_1,x_2)} \min [\mu_{A_1}(x_1),\ \mu_{A_2}(x_2),\ \mu_R(x_1,x_2)] \qquad (21)$$

where $\mu_R(x_1, x_2)$ represents the specified additional

restriction on any given pair of values x_1 and x_2 .

Composition

Given a fuzzy relation between two fuzzy variables, one can obtain the fuzzy value of one variable induced by a known value of the other. This is facilitated by the operation of composition which corresponds to the algebraic rule: if x=a and y=f(x), then y=f(a).

If R is a fuzzy relation in X x Y and A is a fuzzy set in X, then a fuzzy set B is induced in Y through R (Figure 5) with membership :

$$\mu_B(y) = \underset{x}{\text{Sup}} \min [\mu_A(x), \mu_R(x, y)] \qquad (22)$$

As a simple numerical example, if the fuzzy estimate of the modulus E (kg/ cm^2) of the first layer in the dam example is :

$$E_1 = 0.8/45.0 + 1.0/50.0 + 0.6/55.0$$

then the fuzzy relation shown in Table 2 would yield a fuzzy estimate for the modulus of the second layer as :

$$E_2 = 0.6/40.0 + 0.8/45.0 + 1.0/50.0 + 0.8/55.0$$

More generally, if R and S are fuzzy relations in X x Y and Y x Z respectively:

$$\mu_{R.S}(x,z) = \underset{y}{\text{Sup}} \min [\mu_R(x,y) , \mu_S(y,z)] \qquad (23)$$

where R.S is the composition of R and S.

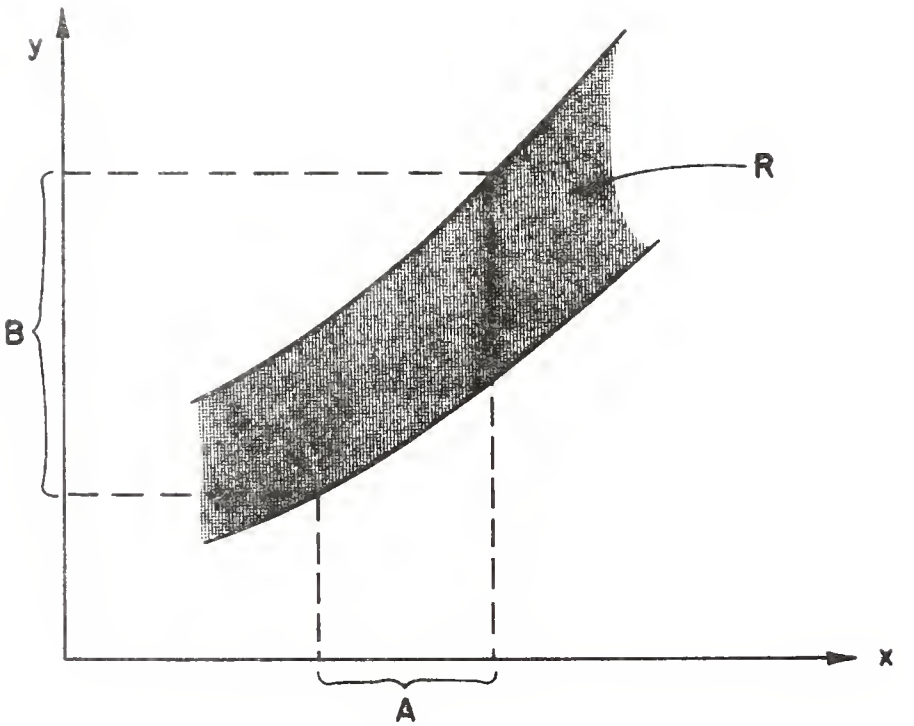


Figure 5. Fuzzy Composition

A more complete definition of the operation of composition is provided in Appendix A for the interested reader. Note that equation (22) is similar to the extension principle in equation (19). Dubois and Prade (1980) state that the extension principle is a special case of composition. The author's proof of this result is also presented in Appendix A.

Applications of fuzzy sets

Since the theory of fuzzy sets provides a tool to deal with problems that cannot be treated effectively with classical or stochastic theories, it has found extensive applications in various fields. Apart from a few exceptions, so far this theory has been applied mainly to scientific areas where man is somewhat involved. Some notable exceptions are the tolerance analysis of electrical circuits (Jain,1976) and the notion of fuzzy events in the theory of measurement of incompatible observables (Prugovecki,1974). Applications in areas where human subjectivity is involved will be very briefly outlined in what follows.

Medical diagnosis

Phippe Smets (1981) modelled the medical diagnostic process by considering diagnostic groups as fuzzy sets and observed symptoms as fuzzy information provided by the patients. Esogbue and Elder (1977) noted that most mathematical models of physicians' diagnoses processes suffer from the inability to incorporate all useful data on the patient. Neglected pertinent information is

intrinsically fuzzy, but is important in describing the patient's health status. They present a model based on fuzzy sets theory, for physician-aided evaluation which is a complete representation of information. Patient's past history, present symptoms, signs observed upon physical examination, and results of clinical and diagnostic tests, are represented by fuzzy matrices. They formulate a general fuzzy decision model based on fuzzy clustering theory, for an optimal diagnosis by a physician.

Structural damage assessment

Several workers, Blockley (1977), and Munro and Brown (1983) realized that the actual number of structural failures is orders of magnitude higher than that predicted by reliability analysis. They agree that this is a consequence of unanticipated gross errors in construction, effects and severities of which can only be evaluated using linguistic terms. Concepts of fuzzy relations, composition and fuzzification were incorporated to evaluate the influence of these errors on the original safety measure of the structure. Ishizuka et. al (1981) introduced a rule-based inference method for damage assessment. A characteristic feature of a rule-based inference approach is the ability to collect a variety of knowledge in small fragments and then use them to solve a complex problem. While representing linguistic damage classes by fuzzy sets, Ishizuka et al (1981) also show that inferences with fuzzy reasoning can conveniently be handled using concepts of fuzzy sets theory.

Economics

Ponsard (1981) applies fuzzy sets theory to analyse consumers' spatial preferences. He discards the usual assumption of consumers' capability of perfectly discriminating between goods, in the classical theory of consumer behaviour. The notion of fuzzy consumer preferences of goods is introduced along with the definition of a fuzzy utility as a numerical representation of the imprecise preference.

Psychology

According to Kochen (1975), this theory can offer psychology new concepts to use as building blocks for improved theories and in return, psychology can offer fuzzy sets theory not only continuing challenges and test problems but methods of experimentation as well. He designed experiments among college students to find out how they interpret words such as "far", "more or less far", "very far" etc., in a specific context. One purpose of such experiments was to relate human cognition of such variables with possible membership functions (Kochen and Badre ,1976). Since the results showed that many people interpret such variables as fuzzy sets, he concludes that fuzzy sets theory seems appropriate for conceptualizing certain aspects of behaviour of, perhaps, half the population.

This theory has also been widely applied in fields such as artificial intelligence and robotics, image processing and pattern recognition as well as in control systems. The reader is

directed to Dubois and Prade (1980) where a volume of references are listed on these and many other applications. Recent applications are also discussed by Zadeh (1984).

CHAPTER 3 : PAVEMENT EVALUATION 1 - RIDEABILITY

Pavement serviceability rating

Of the pavement characteristics, roughness has the greatest impact on rideability. Thus, in many states the initial screening of road sections for determining maintenance needs is on a roughness basis.

The PCA roadmeter is used in Indiana for roughness measurements at the network level. These are converted to a pavement serviceability index (PSI) scale for rideability performance levels (Carey and Irick, 1960). The conversion is made with regression equations developed by statistical correlation of roadmeter readings and pavement serviceability ratings (PSR) for a sample of pavements in the state. Pavement serviceability ratings are given by a panel of 20 road users of different ages and backgrounds who are instructed to rate each of these selected pavements on a scale of 0-5, according to their evaluation of its rideability. Because the PSR of a particular road section is obtained as the mean of the panel ratings, it is clear that individual ratings are treated as random variables. Judgments of the panel members are subjective and certainly involve more human uncertainty than random uncertainty. Moreover, no distinction is made between the ratings of people with different professional backgrounds.

Fuzzy logic can be used to develop a PSR which reflects the human uncertainty as well as the relative perceptiveness (significance) of various panel members in judging the importance of the sources of roughness in pavement performance. The fuzzy PSR allows one to take advantage of such perceptiveness in the rating panel.

In this chapter, the techniques to formulate a fuzzy PSR are analyzed. As a first step, the rating panel is separated into groups of individuals with similar backgrounds, to account for differences in perceptiveness. Furthermore, to avoid any differences within a group such as experience, age, etc., each group can also be subdivided into a sufficient number of subgroups. One possible division made in consultation with highway engineers, for a rating panel used by the Research and Training center of the Indiana Department of Highways is shown in Table 3. The individual opinions are then aggregated using different weights and suitable aggregation techniques at the subgroup and group levels, until the panel rating is formed. Then relationships are developed between pavement serviceability rating (PSR) and Roadmeter reading using two distinct approaches.

Vagueness of rater opinion

This approach considers each individual decision (rating) to contain some vagueness (uncertainty). Consider the i^{th} subgroup of any group A and let c_{ij} be the rating of its j^{th} member on the PSR scale. Since his judgment is subjective, this rater really

Table 3. A grouped rating panel

	A Group		B Group		C Group	
	Profession	Age	Profession	Age	Profession	Age
	Research Eng.	50-59	Engineering Asst.	50-59	Lab. Technician	20-29
Major	Research Eng.	50-59	Eng. Asst. Super.	30-39	Mechanic	30-39
					Electronics Technician	40-49
	Research Eng.	30-39	Eng. Assistant	30-39	Training Off.	40-49
Minor	Research Eng.	20-29	Eng. Assistant	20-29	Secretary	30-39
	Research Eng.	20-29	Eng. Assistant	20-29		
			Eng. Assistant	20-29		

says that the rating is the central value in a small domain around c_{ij}. In other words, it is more logical to express this rating as approximately c_{ij}, which can be represented as a fuzzy number A_{ij} as discussed in Chapter 2.

The next step is to select appropriate algebraic functions (L-L type) for the membership function (see fuzzy numbers, Chapter 2). It was pointed out previously that the user should exercise his own judgment together with experts' opinion in choosing the shape of the curve.

Kochen and Badre (1976) performed a set of psychological experiments among college students to relate human cognition of linguistic variables to membership functions, and discovered that a marginal increase of a person's strength of belief that "r is C" (where C is a fuzzy set) is; 1) proportional to the degree of belief that "r is C" and 2) proportional to the degree of belief that "r is not C".

Mathematically this can be expressed as :

$$d(\mu_C(r))/dr \propto \mu_C(r) \tag{24}$$

$$d(\mu_C(r))/dr \propto [1 - \mu_C(r)] \tag{25}$$

This is also known as the implicit analytical definition of membership functions. As shown in Appendix A, these two conditions are sufficient to select the needed L function as :

$$\mu(r) = L(r) = \frac{e^{kr+c}}{1 + e^{kr+c}} \tag{26}$$

where c and k are constants to be determined from boundary

conditions. Two boundary conditions are available to determine k and c (Figure 6) :

Since the central value has the maximum membership ;

$$\mu = 1.0 \quad \text{for} \quad r = c_{ij} \tag{27a}$$

Since the extreme value has zero membership ;

$$\mu = 0.0 \quad \text{for} \quad r = c_{ij} - r_{ij} \tag{27b}$$

However, according to the exponential nature of equation (26), $\mu = 1.0$ for $r \to \infty$ and $\mu = 0.0$ for $r \to -\infty$. This problem can be overcome by modifying the boundary conditions to :

$$r = c_{ij} \quad \text{for} \quad \mu = 1.0 - \delta \tag{28a}$$

$$r = c_{ij} - r_{ij} \quad \text{for} \quad \mu = \delta \tag{28b}$$

where δ is a small numerical quantity. A typical L curve obtained for a value of δ of 0.01 is given in Figure A (Appendix A).

Zadeh (1975) introduced an L-L function known as a π curve (Figure 6) where the L function is defined by :

$$\mu(r) = 2\left[\frac{r - (\gamma - \beta)}{\beta}\right]^2 \quad \text{for } r \leq (\gamma - \beta/2) \tag{29a}$$

$$\mu(r) = 1 - 2\left[\frac{r - \gamma}{\beta}\right]^2 \quad \text{for } \gamma \geq r \geq (\gamma - \beta/2) \tag{29b}$$

where γ and β are two parameters which may be adjusted to fit specified boundary conditions. It is obvious that the present case satisfies the following boundary conditions.

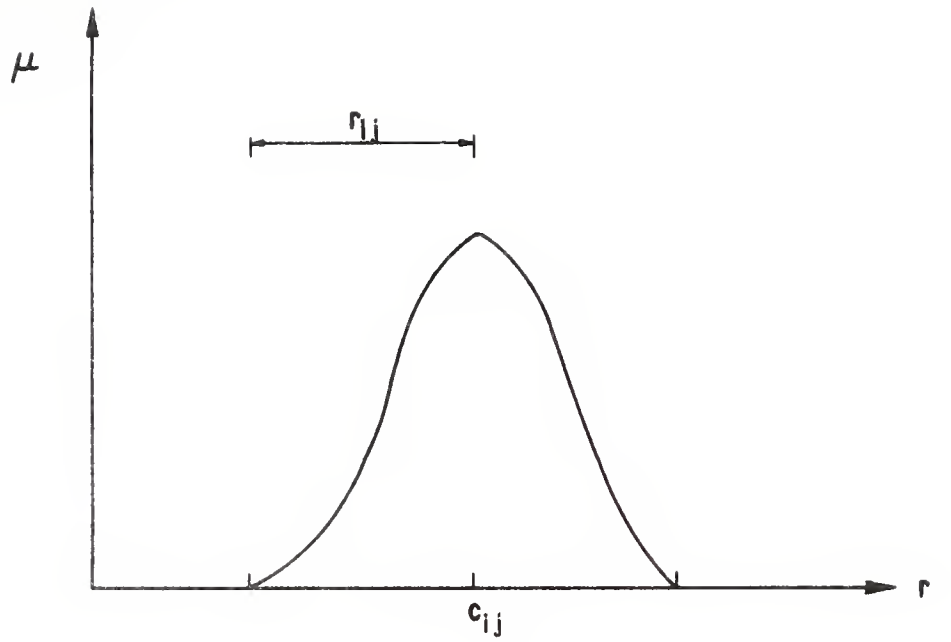


Figure 6. 'π' Curve

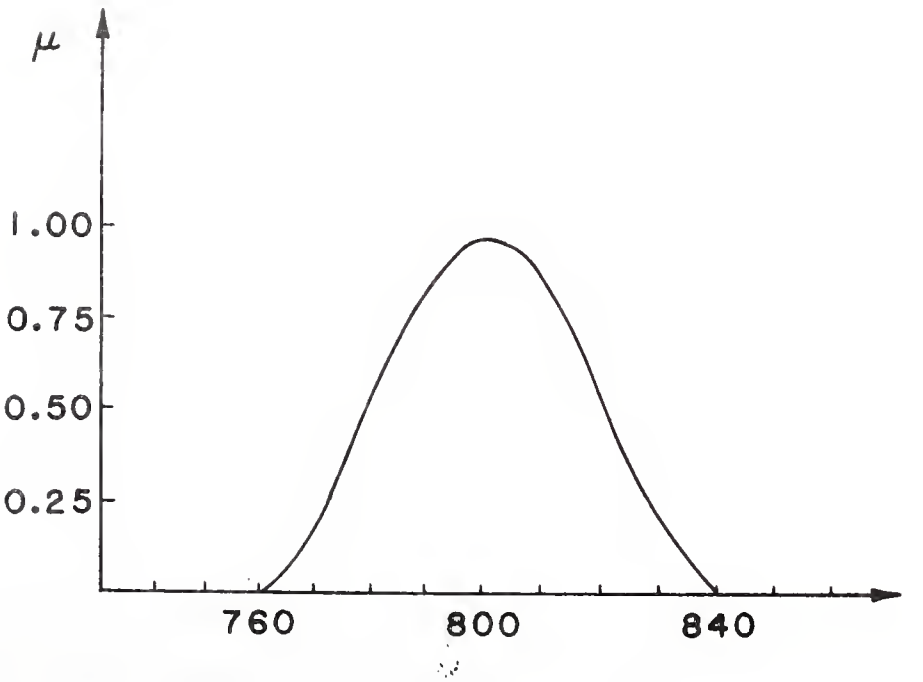


Figure 7. Fuzzified roadmeter reading

$$\gamma = c_{ij} \tag{30a}$$

$$\beta = r_{ij} \tag{30b}$$

The latter L function is also plotted in Figure A (Appendix A) in dashed lines to show the close agreement between the two types of L functions. The problem of estimating r_{ij} for the present case will be addressed later.

Because of their analytical simplicity it is preferred to use L functions of π type (equations (29a) and (29b)) to work with fuzzy numbers. As has been shown, one of their features is the very close agreement with exponential curves resulting from the implicit definition of membership functions. However, research has also been done with fuzzy sets using linear membership functions as well (Tanaka,1982).

Formation of subgroup and group opinions

When the opinion A_{ij}, of each member in the subgroup i, is represented as a fuzzy number, aggregation can be performed for all members in subgroup i by the union of all such opinions:

$$A_i = \bigcup_i A_{ij} \tag{31}$$

A_i is the opinion of the i^{th} subgroup of group A. This process can be repeated to form other subgroup opinions.

Aggregation then can be extended to combine all subgroups of group A, while accounting for the relative weight (w_i) assigned to each subgroup with respect to the others in group A. With the

introduction of the relative weights w_i, the combined opinion of the group A and memberships are obtained as follows:

$$A = \underset{i}{U}\, w_i\, A_i \quad \text{where} \quad \sum_i w_i = 1 \tag{32}$$

$$\mu_A(x) = \max_i \left[w_i \times \mu_{A_i}(x)\right] \tag{33}$$

Equations (31) and (32) define the same type of aggregation (union), the only difference being the introduction of weights in equation (32). Modification of the union operation to include weights (equations (32) and (33)) is defined by Zadeh (1973) as the operation of dimunition. Within any subgroup i, it is assumed that there is no difference in perceptiveness; thus, weights are not introduced in aggregating individual rater opinions in equation (31).

It was pointed out in Chapter 2 that the operations of union and intersection can also be employed in aggregating fuzzy sets. Appropriateness of either one of the operations of union and intersection depends on the particular situation. It is obvious from Figure (3b) that intersection is not suitable for gathering information, especially when there is a lack of agreement between the sources. This is exactly the case when individual and subgroup opinions are gathered. On the other hand, union (Figure 3a) is less restrictive and is useful in acquiring as much information as possible from all sources ; no opinions are discarded. Hence, the union operation is appropriate at the initial stages of the PSR formulation (equations (31) and (32)).

As an example, let us assume that the panel members have been divided into two groups, A of highway engineers, and B of laymen (raters who are not highway engineers). Among the group of highway engineers, two subgroups have been identified: A_1 of experienced engineers; and A_2 of engineers with little experience. Similarly, the group of laymen is divided into two subgroups; B_1 and B_2, of frequent and infrequent road users, respectively. Further, let us assume that each subgroup opinion of the quality of a given pavement is represented by the fuzzy sets below:

$A_1 = 0.4/2.7 + 0.5/2.8 + 1.0/2.9 + 0.9/3.0 \quad w_1 = 0.6$

(experienced)

$A_2 = 0.6/2.8 + 0.8/2.9 + 1.0/3.0 + 0.9/3.1 \quad w_2 = 0.4$

(little experience)

$B_1 = 0.8/2.8 + 1.0/2.9 + 0.7/3.0 + 0.5/3.1 \quad w_1 = 0.7$

(frequent road users)

$B_2 = 0.9/2.8 + 1.0/2.9 + 0.9/3.0 \quad w_2 = 0.3$

(infrequent road users)

According to the notation adopted for fuzzy sets, 0.4, 0.5, 1.0 and 0.9 are the degrees of support of the subgroup A_1 for the values 2.7, 2.8, 2.9 and 3.0, respectively. In other words, members of the subgroup A_1 indicate a range of values (2.7-3.0) as their PSR, the most favored value being 2.9. Subgroups A_1 and A_2 consist of people with similar backgrounds, but their perceptiveness is assumed to differ due to differences in experience.

Experienced engineers may have a deeper insight into the road condition, so far as the maintenance requirements are concerned. Thus, by introducing the relative weights w_1 and w_2 ($w_1 > w_2$), their opinion is given more weight in arriving at the combined opinion of group A. Similar remarks apply to subgroups B_1 and B_2.

According to equations (32) and (33), the group opinions A and B can be obtained as:

$$A = 0.24/2.7 + 0.30/2.8 + 0.6/2.9 + 0.54/3.0 + 0.36/3.1$$

$$B = 0.56/2.8 + 0.7/2.9 + 0.49/3.0 + 0.35/3.1$$

However, the fuzzy sets A and B have lost their normality (maximum membership value is not equal to 1.0) due to the subgroup weighting, and therefore, are not on a common base for continued combinations. To prevent any group from suffering due to internal weighting both of them should be normalized as:

$$A = 0.4/2.7 + 0.5/2.8 + 1.0/2.9 + 0.9/3.0 + 0.6/3.1$$

$$B = 0.8/2.8 + 1.0/2.9 + 0.7/3.0 + 0.5/3.1$$

Aggregation of different group opinions

When the separate group opinions are formed, they can be combined to obtain the PSR of the section. The following desirable features guide the selection of the aggregation techniques to be used at this stage of the manipulation :

(1) The section PSR should be as precise as possible,

covering as narrow a range as possible ;

(2) It is desirable for the final PSR to contain information agreeable to all groups ;

(3) Group opinions formed on the basis of collecting information will, in general, overlap with one another. Thus, much information is present in the regions of overlap.

The foregoing suggests that a "pessimistic" or more restrictive type of aggregation is preferred at this stage (Figure 3a). There are two such operations, intersection and algebraic product. Intersection imposes a strong (maximum) restriction through the "min" operator, thereby eliminating some significant opinions. On the other hand, algebraic product is less restrictive and interacts between different opinions to retain every judgment since it does not follow fuzzy logic. The latter was found to be more appropriate at this stage, and reasons for this choice will be explained later in the discussion.

The operation of attaching weights to opinions of subgroups and groups is based on the fact that different groups have different insights into the serviceability of pavements. For instance, highway engineers likely view the inadequacies and hazardous areas of a road from a technical, or a maintenance urgency, point of view, whereas laymen likely are only concerned about the riding comfort. In view of this, the attachment of different importance levels to each group in accordance with the groups perceptiveness is proposed. This is the second

modification proposed to the conventional approach. Attachment of relative weights to each subgroup in a particular group also stems from this reasoning.

For the pessimistic combination of unequally important opinions, the following rule was proposed by Yager (1978) :

$$\text{Combined opinion} = \bigcap_i G_i^{\alpha_i} \tag{34}$$

where the index α_i denotes the importance of the group opinion G_i. The operation $G_i^{\alpha_i}$ produces a concentration, or a dilation, of information in G_i. If the exponent α_i is between 1.0 and 2.0 this operation "concentrates" the information contained in the membership function around the point of maximum membership. If α_i is between 0.0 and 1.0, the membership function is dilated. Concentration and dilation are useful in making provision for the relative importance of an individual's opinion and judgment (Elms,1981 and Gunaratne et al., 1984), as well as in handling linguistic hedges such as "very economical", "more or less safe", etc.(Zadeh, 1974). If equation (34) is used to obtain the PSR, the corresponding membership is found using :

$$\mu_{G_i^{\alpha}} = [\mu_{G_i}]^{\alpha} \tag{35}$$

It is realized that the operation in equation (35) is more interactive than fuzzy. Therefore it seems more meaningful to replace the intersection in equation (34) with the algebraic product. This gives (with equations (9a) and (35)) :

$$\mu_{PSR} = \prod_i \mu_{G_i}^{\alpha_i} \tag{36}$$

This makes it possible to give a physical interpretation of exponentiation in the following manner. To look after the differences in importance of the groups, n_i number of groups G_i can be introduced in place of one G_i, with n_i being an indicator of the importance of G_i. For example if $n_1 = 3$ and $n_2 = 1$, this means that in obtaining a final decision, three opinions similar to G_1 are used against one opinion similar to G_2. Then, performing the aggregation with the algebraic product (equation 9a) gives :

$$\mu_{agg.} = [\mu_{G_1} \text{..upto } n_1] \ldots [\mu_{G_i} \text{..upto } n_i]$$

$$= \prod_i [\mu_{G_i}]^{n_i} \tag{37}$$

which resembles the concentration operation in equation (36).

A constant λ can be found such that,

$$n_i / \lambda = \alpha_i \tag{38}$$

where $0.0 \leq \alpha_i \leq 2.0 \qquad \forall\, i$ (39)

which satisfies conditions for concentration and dilation.

From equations (36),(37) and (38) it is seen that α_i can be visualized as physically representing multiples of G_i opinions, normalized on a scale 0.0-2.0. In contrast, the factors w_i are weights to be attached to subgroup opinions, normalized on a scale of 0.0-1.0. From the above discussion, it is intuitively seen that the former creates a more marked discrimination between different opinions than that created by a mere weighting.

Since equations (36) and (37) show that concentration is an interactive and not a fuzzy operation, it is logical to use

equation (36) (algebraic product) in preference to equation (34). As an example, the above groups A and B can now be concentrated or dilated according to each group's relative significance. Let us assume that group A opinion is very important and that of B is not so important, and assign $\alpha = 2.0$ and $\beta = 0.5$, respectively. The concentrated form of A is :

$$A^* = A^{\alpha}$$

$$= 0.16/2.7 + 0.25/2.8 + 1.00/2.9 + 0.81/3.0 + 0.36/3.1$$

and the dilated form of B is:

$$B^* = B^{\beta}$$

$$= 0.89/2.8 + 1.00/2.9 + 0.83/3.0 + 0.71/3.1$$

Finally, using equation (36),

$$PSR = A^*.B^* = A^{\alpha}.B^{\beta}$$

$$= 0.22/2.8 + 1.00/2.9 + 0.67/3.0 + 0.26/3.1$$

This is the "fuzzy" PSR for the pavement section under consideration. The PSR of a section originates from subjective judgments which support a region of values rather than a single value. The conventional PSR is a discrete number and thus does not indicate this region of PSR, supported by the members of the panel. On the other hand, the fuzzy PSR shows this region of support as well as the degree of support for each value. In addition, it incorporates each individual's perceptiveness of pavements while carrying his judgment up to the final stage of the analysis. These advantages show the proposed fuzzy PSR to be

an improvement over the conventional determination.

The group importance factors, α_i, depend on each group's perceptiveness of how the highway performs and how the sources of the roughness influence that performance. Similar remarks apply to subgroup factors w_i as well. These and the uncertainty in the individual rating (r_{ij}) are to be obtained by consulting highway experts. The author prepared a questionnaire (No. 1 in Appendix F) to obtain the factors denoting the relative significance of possible panel groups and the relative weights for the subgroups. In addition a subjective opinion of the extent of vagueness of individual opinions (r_{ij}) was also sought in this questionnaire. A limited number of responses have been received from experts from the Federal Highway Administration and the Indiana Department of Highways. They serve as a basis for the numerical examples presented in the dissertation. Further information is being sought by another research associated with this project to provide the data base and expert knowledge necessary for implementation of the concepts proposed herein.

Imprecise importance factors

In the previous section it was shown that fuzzy sets can effectively model human uncertainty and also incorporate professional judgment in decision making. It is realized that since the factors α_i represent experts' collective judgment, they may not be precisely defined parameters and may be fuzzy sets. The possibility of having fuzzy α_i can be incorporated in the

operations of concentration and dilation using Type 2 fuzzy sets. It is shown herein that Type 2 fuzzy sets are encountered when the decision making process involves concentrating or dilating professional judgment with fuzzy α_i factors.

The concept of fuzzy sets of Type 2 has been proposed as an extension of ordinary fuzzy sets for such cases where the membership function is itself a fuzzy set, i.e. the "grade of membership" or "degree of belief" is fuzzy (Zadeh, 1974). Type 2 fuzzy sets are intuitively appealing for engineering applications as, very often, it is difficult to assign or develop crisp memberships.

Mathematically, Type 2 fuzzy sets are those for which the function $\mu_A(x)$ is not a direct mapping of the space X on the interval [0,1] (Chapter 2), but is itself a fuzzy set; they can be represented as follows :

$$A = \bigcup_i \mu_A(x_i)/x_i \qquad x_i \in X \tag{40a}$$

with

$$\mu_A(x_i) = \bigcup_j f(u_j)/u_j \qquad u_j \in [0,1] \tag{40b}$$

In this latter equation, the function f(.) is also a membership function which, to each value u_j, associates a number in the interval [0,1].

Although Type 2 fuzzy sets are not in frequent use, the basic operations for this class of sets have been defined (Mizumuto and Tanaka, 1976). In particular, the union and

intersection operations of ordinary sets have been extended to form the "Join" and "Meet" operations, respectively, as follows :

Join: If A and B are Type 2 fuzzy sets with

$$\mu_A(x) = \underset{i}{U}\ f(u_i)/u_i$$

$$\text{and } \mu_B(x) = \underset{j}{U}\ g(v_j)/v_j,$$

then C = A U B is a Type 2 fuzzy set with

$$\mu_C(x) = \underset{i,j}{U}\ f(u_i) \wedge g(v_j)\ /(u_i \vee v_j) \tag{41}$$

where $\wedge$ and $\vee$ denote "min" and "max", respectively.

Meet: D = A B is also a fuzzy set with

$$\mu_D(x) = \underset{i,j}{U}\ f(u_i) \wedge g(v_j)\ /(u_i \wedge v_j) \tag{42}$$

The extended algebraic product can also be defined as :

$$\mu_{A.B} = \underset{i,j}{U}\ f(u_i) \wedge g(v_j)/(u_i . v_j) \tag{43}$$

The operations of concentration and dilation of a fuzzy set A can be written as:

$$A^{\alpha} = \underset{i}{U}\ \mu(x_i)^{\alpha}/x_i \tag{44}$$

The parameter α is an index which reflects the level of importance and relative significance of an individual or a group of individuals (equation 34).

By using the definition of the exponential and scalar multiplication of fuzzy numbers (Dubois and Prade, 1980), it can be shown that if the exponent α is also a fuzzy set with a

membership function given by the expression :

$$\alpha = \underset{j}{U}\ f(y_j)/y_j \tag{45}$$

Then A^{α} is a fuzzy set of Type 2 :

$$A^{\alpha} = \underset{i}{U}\ \mu_i/x_i \tag{46a}$$

where

$$\mu_i = \underset{j}{U}\ f(y_j)/[\mu(x_i)]^{y_j} \tag{46b}$$

A mathematical proof of this result is given in the Appendix B. As an illustration, consider the fuzzy opinions of groups A and B in the previous example. To avoid the numerical complexity arising from a large number of terms, let us consider only the terms with relatively high memberships :

engineers $A = 1.0/2.9 + 0.9/3.0 + 0.6/3.1$

laymen $B = 0.8/2.8 + 1.0/2.9 + 0.7/3.0$

Let us assume that instead of the values 2.0 and 0.5 used in the previous section, the indices α and β are imprecise and the following fuzzy sets are required to express the highway experts' opinion regarding the importance of each group :

$\alpha = 0.6/1.5 + 0.8/2.0$

$\beta = 0.8/0.5 + 0.6/1.0$

It should be mentioned here that α and β fuzzy sets can be directly obtained from a survey of expert responses by the method used to obtain acceptable serviceability ratings (Chapter 5).

By applying equations (46a) and (46b), the concentrated/dilated Type 2 fuzzy sets shown in Table 4 are obtained. These modified group opinions can now be aggregated using the algebraic product operator (equation (43)). The resulting PSR , which is also a Type 2 fuzzy set, is shown in Table 5. Although somewhat more complex than the ordinary fuzzy PSR, a physical interpretation of this fuzzy PSR is possible. For instance, Table 5 shows that there is combined panel support for 2.9 and 3.0 as the PSR of this pavement section. The degree of belief in 2.9 is 1.0, whereas that in 3.0 is between 0.6 and 0.71 with more support surrounding 0.68 .

Variability of the Roadmeter reading

The next step is to correlate the PSR with Roadmeter measurements to obtain a PSR-RR relationship for the highway network. In Indiana, a PCA Roadmeter is used to evaluate pavement roughness at the network level. This Roadmeter must be calibrated from time to time using a set of rated pavement sections. It is known that there is an inherent variability associated with the Roadmeter. Its mechanism records the cumulative vertical movement along the pavement profile, which is a measure of the roughness of the section. The Roadmeter count depends on the exact path traced by the vehicle; thus, repeated measurements exhibit a scatter. This is characteristic of imprecise measurements. Representing such a reading by a fuzzy number can be more meaningful.

Table 4. Concentration and Dilation of Opinions

PSR	2.8	2.9	3.0	3.1
$A^* = A^{\alpha}$		0.8/1.00 + 0.8/0.81	0.6/0.85 + 0.8/0.36	0.6/0.47
$B^* = B^{\beta}$	0.8/0.89 + 0.6/0.8	0.8/1.0	0.8/0.84 + 0.6/0.7	

Table 5. Fuzzy PSR

PSR	2.9	3.0
PSR= $A^{\alpha}B^{\beta}$	0.80/1.0	0.60/0.71 + 0.80/0.68 0.6/0.6

This idea can be further clarified in the following manner. Assume that an actual representative roughness of the section, X, exists which cannot be precisely identified by the roadmeter because it produces different counts on repeated passes. When only one Roadmeter reading, x, is available, the statement "X is approximately x" can be made. Therefore as far as the Roadmeter is concerned, X, the actual roughness, is a fuzzy number with x taking the maximum membership.

Another concern of the Indiana Department of Highways is the influence of the following factors on the Roadmeter reading : (1) gas tank level; (2) climatic changes, and (3) driver characteristics. Although these factors may cause a significant variation, present statistical models have not been able to incorporate them successfully. One reason for this is the difficulty in quantifying these effects. In view of this, it is more convenient to think of these effects as imprecisions, or vagueness induced by the above factors on the reading. This further supports the use of a fuzzy number to represent the Roadmeter reading.

The theory of fuzzy sets provides an effective means of solving problems where imprecision or system uncertainty occurs, provided that an estimate of the extent of variation is available. Prugovecki's (1974) representation of incompatible measurements by fuzzy sets, and Jain's (1976) fuzzy tolerance analysis are two examples where imprecise measurements are fuzzified. Prugovecki (1976) stated that measurement of observables in quantum mechanics, such as the measurement of a microsystem,

carried out on a macroscopic level, do not yield sharply localized results. Thus, in his work he adopted the postulate that such measurements can be called fuzzy events represented by normalized fuzzy sets. Then he presented fuzzy probabilistic concepts which can be used for the determination of membership functions for such fuzzy sets. Jain (1976) incorporated the tolerance interval of a measured resistance, in electrical circuit analysis, by treating the resistance as a fuzzy set. On using a constant membership function within the tolerance interval for the fuzzy resistance, he used fuzzy sets techniques for the circuit analysis. Following along these lines, fuzzification of Roadmeter readings is elaborated in the next section as one approach to handle its inherent variability.

Fuzzification of Roadmeter reading

In representing a reading by a fuzzy number, the fuzzification technique suggested by Zadeh (1973) can be adopted. First, all the factors contributing to the imprecision are identified. Then, a kernel of fuzzification is formed for each factor according to the extent of variation it causes. Such a kernel is a fuzzy set having a range of support equal to the possible variation caused by that factor.

When a kernel, K(x), defined by a fuzzy set :

$$K(x) = U\ \mu_{K(x)}(y)/y \tag{47a}$$

is operated on the element x of a fuzzy set A where,

$$A = U\ \mu_A(x)/x \tag{47b}$$

it increases the vagueness of the fuzzy set and produces a "fuzzier" set :

$$F = U\ \mu_F(y)/y \tag{47c}$$

where

$$\mu_F(y) = \mu_A(x).\mu_K(y) \tag{47d}$$

As an example, if the observed scatter of the Roadmeter count due to inability of repetition is 30% of the reading x, one possible kernel that will represent this uncertainty is:

$$K_R(x) = 0.0/0.85x + 0.34/0.9x + 0.66/0.95x + 1.0/x + 0.66/1.05x + 0.34/1.1x + 0.0/1.15x$$

where the membership function is assumed to be linear. If a roadmeter count of 1000 is measured, it can be written as a crisp fuzzy set:

$$A = 1.0/1000$$

and, operating K_R on A produces :

$$F = 0.0/850 + 0.34/900 + 0.66/950 + 1.0/1000 + 0.66/1050 + 0.34/1100 + 0.0/1150$$

This shows that a Roadmeter reading of 1000, can actually indicate any value between 850 and 1150 due to possible variations observed in repeated measurements, with the highest degree of support being on the measured value.

Let $K_G(x)$, $K_D(x)$, and $K_V(x)$ be kernels associated with variations of gas tank level, driver characteristics and climate

respectively, for any Roadmeter count x. The four kernels can be operated successively on the measured value x to derive a fuzzy Roadmeter count, which includes the variability due to all of the relevant factors. Several such operations can become cumbersome and would involve excessive computer time. This problem can be avoided by forming one single kernel out of all the constituent kernels:

$$K(x) = K_R(x) \cdot K_C(x) \cdot K_D(x) \cdot K_S(x) \qquad (48a)$$

K(x) can operate on any reading as a single operator. Using basic fuzzy algebra it can be shown that such a single operation does not change the final result. The following example illustrates the successive fuzzification procedure.

Let us assume that the roughness measured on a certain pavement is 2000. Further, assume for simplicity that the maximum variation in readings for the same pavement section on repeated passes is 7.5% and the maximum variations due to the changes in gas tank level, driver characteristics and climate are 5.0% each. The above information can be obtained either by examining past data, or by consulting highway experts.

It is possible to assign a membership function for the kernel of a continuous variable such as the Roadmeter reading, using a proper algebraic function. In practice, discretization into a number of specified intervals is neccessary, especially when memberships are manipulated with the computer. Suppose such an

interval of 1.25% is chosen in the present case, and the following fuzzy kernels are formed based on π curves (equation (29)).

$$K_R(x) = 0.22/0.975x + 0.78/0.9875x + 1.0/x + 0.78/1.0125x + 0.22/1.025x$$

$$K_G(x) = K_D(x) = K_V(x) = 0.5/0.9875x + 1.0/x + 0.5/1.0125x$$

Then the combination of kernels can be done as follows:

$$K_V(x).K_D(x) = [0.5/0.9875x + 1.0/x + 0.5/1.0125x] [0.5/0.9875x + 1.0/x + 0.5/1.0125x]$$
$$= [0.25/0.975x + 0.5/0.9875x + 1.0/x + 0.5/1.0125x + 0.25/1.025x]$$

Proceeding in this manner leads to :

$$K(x) = K_V(x) \cdot K_D(x) \cdot K_G(x) \cdot K_R(x)$$
$$= 0.03/0.9375x + 0.10/0.95x + 0.20/0.9625x + 0.39/0.975x + 0.78/0.9875x + 1.00/x + 0.78/1.0125x + 0.39/1.025x + 0.20/1.0375x + 0.10/1.05x + 0.03/1.0625x$$

It is realized that formation of such a combined kernel vastly reduces computer time and storage because the successive fuzzification will not have to be performed everytime a different pavement section is analysed. This combined kernel can now operate on a Roadmeter reading of 2000, to produce:

RR = 0.03/1875 + 0.10/1900 + 0.20/1925 + 0.39/1950
+ 0.78/1975 + 1.0/2000 + 0.78/2025 + 0.39/2050
+ 0.20/2075 + 0.10/2100 + 0.03/2125

This is the final fuzzy roadmeter reading depicting the variations due to the above factors.

The most important stage of the procedure is the formation of the membership functions of the respective kernels. Information on the variations caused by the factors that are known to influence the Roadmeter reading, can be sought from highway engineers and experts who are knowledgeable in these research areas, and have used the Roadmeter extensively. Because the relevant data collected over a time can also be valuable, experts can make use of such documentation in imparting their knowledge. Questionnaires were prepared (Nos. 2 and 3 in Appendix F) to seek information from highway personnel on this new idea of fuzzification of imprecise measurements.

Selecting the shape of the curve to be used is again left to the analyst. It is logical to expect the variation caused by the influencing factors to be symmetrical with respect to the measured value. Therefore, any kernel should produce a fuzzy number with a symmetric membership function when operated on a given measured value. This condition can only be satisfied by a kernel with an L-L type membership function (Figure 4b). To maintain consistency with the work presented so far, π curves (equations 29a and 29b) have been selected to describe membership functions of the different kernels. To express a kernel of fuzzification

by a π curve, equations (29a) and (29b) can be modified to,

$$K = \pi(x, \beta) \tag{48b}$$

where x is the reading to be fuzzified.

PSR - RR relationship

At present, the pavement serviceability rating and the Roadmeter reading are correlated by linear regression analysis since both are considered as random variables. In the proposed approach, both PSR and the Roadmeter reading are expressed as fuzzy variables, and they can be correlated using a fuzzy relationship. (see fuzzy relations, Chapter 2). Equations (10) and (16) can be used to form this relation, with X and Y replaced by PSR and RR (roadmeter reading), respectively. Then, the fuzzy relation can be expressed by:

$$R = PSR \times RR \tag{49}$$

with

$$\mu_R(x_i, y_j) = \min [\mu_{PSR}(x_i), \mu_{RR}(y_j)] \tag{50}$$

When a set of data is available for correlation, such component fuzzy relations are formed for each pair of data. Let us consider the example discussed in the PSR formulation and let the corresponding roadmeter reading for that pavement section be 800. Assuming the range of variation of this reading due to all the relevant factors to be 10%, a simple fuzzification can be performed on this reading (Figure 7):

RR = 0.125/770 + 0.50/780 + 0.875/790 + 1.0/800 + 0.875/810
+ 0.50/820 + 0.125/830

Since the fuzzy PSR was expressed by :

PSR = 0.22/2.8 + 1.0/2.9 + 0.67/3.0 + 0.26/3.1,

the binary fuzzy relationship between PSR and RR is formed according to equation (50). As an example, μ_R (3.0, 790) is 0.67, the minimum of μ_{PSR} (3.0) = 0.67 and μ_{RR} (790) = 0.875. Repeating this operation for each set of values results in Table 6. Membership values of a fuzzy relation are analogous to the strength of the link between the corresponding RR and PSR values. For example, RR = 800 and PSR = 2.9 are strongly linked (membership values = 1.0) whereas RR = 830 and PSR = 3.1 are weakly related (membership value = 0.26).

Such relationships can be formed for all the sample sections, covering wide ranges of PSR and RR. Let R_k denote such a relation with a membership function $\mu_{R_k}(x_i,y_j)$ obtained from the k^{th} sample section. For a given pair (x_i,y_j), different memberships will result from relations formed for different sections. In forming the resultant membership for the pair (x_i,y_j), membership components from all pavement sections must be summed up. Thus, according to fuzzy logic, the union operator can be used as follows :

$$\mu_R(x_i,y_j) = \bigcup_k \mu_{R_k}(x_i,y_j) \qquad (51)$$

This results in the global PSR-RR relationship for the highway network.

Table 6. Fuzzy PSR– roadmeter reading relationship.

PSR \ RR	770	780	790	800	810	820	830
2.8	0.125	0.22	0.22	0.22	0.22	0.22	0.125
2.9	0.125	0.50	0.875	1.0	0.875	0.50	0.125
3.0	0.125	0.50	0.67	0.67	0.67	0.50	0.125
3.1	0.125	0.26	0.26	0.26	0.26	0.26	0.125

PSR - RR data for the highway network, available in the form of a global fuzzy relationship, can now be used to determine the pavement serviceability index (PSI) for a particular pavement section, knowing the roadmeter reading. This can be achieved by the composition operation described in Chapter 2. Given the PSR-RR global relation R, the PSR induced by a known fuzzified roadmeter reading RR is found by :

$$PSI = RR.\ R \tag{52}$$

$$\mu_{PSI}(x) = \sup_{y} \min\ [\mu_{RR}(y), \mu_{R}(x,y)] \tag{53}$$

where PSI is defined as the pavement serviceability index of the section.

As an example, suppose that the fuzzified form of the roadmeter reading for a different pavement section is given by:

$$RR' = 0.6/810 + 1.0/820 + 0.6/830$$

The corresponding fuzzy PSI for this section is obtained according to equation (53) by composing RR' with the fuzzy relation in Table 6, which is assumed to be the global relationship for simplicity.

$$PSI = 0.22/2.8 + 0.6/2.9 + 0.6/3.0 + 0.26/3.1$$

Given the roadmeter reading, the PSI of any pavement section depends on the strength of correlation between PSR and roadmeter readings of all pavement sections. Thus, the global relationship must be used in the above composition, which involves more complicated calculations. A computer program ROAD has been written

for this purpose and is used in the numerical example in Chapter 6 where further illustrations are made.

A different line of thought exists maintaining that a recorded single reading is not proper to be fuzzified. This means that any variation has to be treated as a random variation. In view of this, an alternative procedure to combine the PSR with the roadmeter reading was formulated. In this approach the latter is treated as a random variable while retaining the fuzzy form of PSR. The relationship between RR and PSR is obtained through fuzzy linear regression (Tanaka et al, 1982).

PSR - RR regression

Information is usually available in two forms, namely exact (crisp) information and vague (fuzzy) information. Therefore, more than one technique is required to correlate two different sets of quantities which are known to depend on each other.

In statistical theories, relationships are obtained between two crisp information sets using regression analysis. As shown in Chapter 2 and earlier parts of this chapter, techniques are available in fuzzy sets theory to form relationships between fuzzy information sets. Recently, a fuzzy regression procedure was introduced by Tanaka et al (1982), whereby a set of crisp information can be correlated to a set of fuzzy information. This technique will be used here in correlating the Roadmeter reading, considered as a random variable, to the fuzzy pavement

serviceability rating (PSR) of a pavement section.

If $X_1, X_2, \ldots\ldots, X_n$ are n independent random variables to be correlated to fuzzy variable $\tilde{Y}$, the linear regression equation will have the form

$$\tilde{Y} = \tilde{A}_1 X_1 + \tilde{A}_2 X_2 + \ldots\ldots \tilde{A}_n X_n \qquad (54)$$

where $\tilde{A}_i$ are the fuzzy coefficients to be determined by the regression analysis. If it is assumed that the fuzzy coefficients $\tilde{A}_i$ are symmetric and can be described by π curves (Figure 8a), it can be shown (Appendix C) that for known X_i the fuzzy set for $\tilde{Y}^*$ generated by equation (54) also has a π form (Figure 8b), where :

$$\gamma = \gamma_1 x_1 + \ldots\ldots \gamma_n x_n \qquad (55)$$

$$\beta = \beta_1 x_1 + \ldots\ldots \beta_n x_n \qquad (56)$$

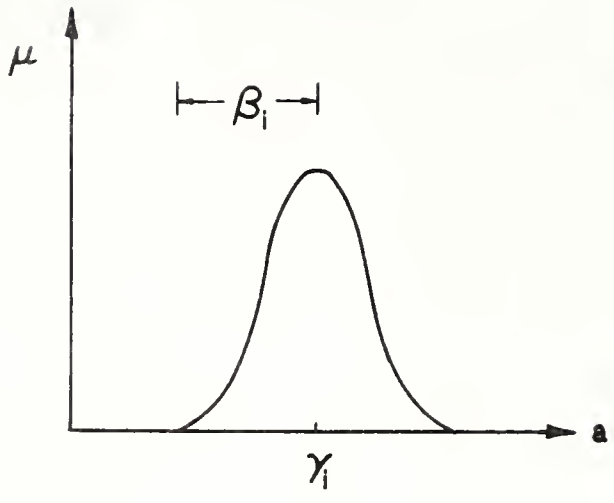
Considering the j^{th} sample of data with input; $x_{j1}, x_{j2} \ldots\ldots x_{jn}$, and output; $\tilde{Y}_j$, the output from the regression model in equation (54) will be

$$\tilde{Y}_j^* = \tilde{A}_1 x_{j1} + \tilde{A}_2 x_{j2} + \ldots\ldots \tilde{A}_n x_{jn} \qquad (57)$$

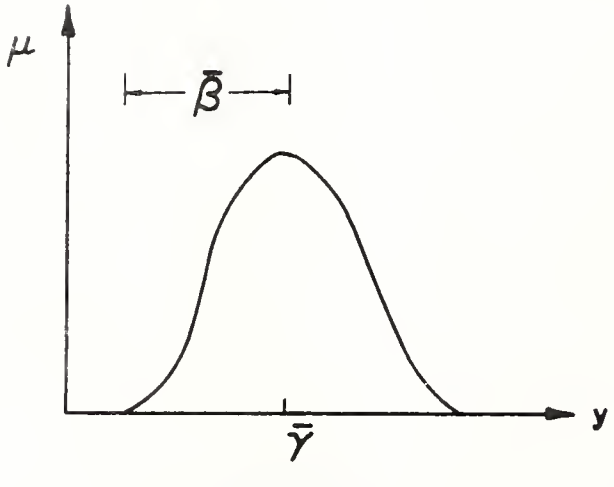
where

$$\gamma = \sum_i \gamma_i x_{ij} \qquad (58)$$

$$\beta = \sum_i \beta_i x_{ij} \qquad (59)$$



a) Fuzzy set $\tilde{A}_i$



b) Fuzzy set Y*

Figure 8. Fuzzy sets $\tilde{A}_i$ and Y*

The objective is to find $\tilde{A}_i(\gamma_i, \beta_i)$ that produces the closest fitting between output $(\tilde{Y}_j^*)$ and the data $(\tilde{Y}_j)$.

The degree of fitting between $\tilde{Y}_j^*$ and $\tilde{Y}_j$ is defined as the maximum h for which the following relation holds :

$$\tilde{Y}_j^h \subset \tilde{Y}_j^{*h} \tag{60}$$

where $\tilde{Y}_j^h$ and $\tilde{Y}_j^{*h}$ are h-cut sets (Dubois and Prade, 1980) defined by :

$$\tilde{Y}_j^h = \{y / \mu_{Y_i}(y) \geq h\} \tag{61}$$

$$\tilde{Y}_j^{*h} = \{y / \mu_{Y^*}(y) \geq h\} \tag{62}$$

The h-cuts are subsets of $\tilde{Y}_j$ and $\tilde{Y}_j^*$ for which the membership values are not less than h. Zadeh also called these sets, h level fuzzy sets. For any data set j, the degree of fitting $(\overline{h}_j)$ is shown in Figure 9. The difficulty in obtaining a closed form solution for $\overline{h}_j$ is apparent and a numerical program was developed to solve for $\overline{h}_j$.

If the desired degree of fitting for the whole set of data (j = 1 ..... N) is H, then the condition of fitting is :

$$\min_j (\overline{h}_j) \geq H \tag{63}$$

The objective of the analysis is to find the values of γ_i, β_i subject to the condition in equation (63). Since an infinite number of solutions may satisfy this condition, an additional condition must be introduced for uniqueness. For this purpose the

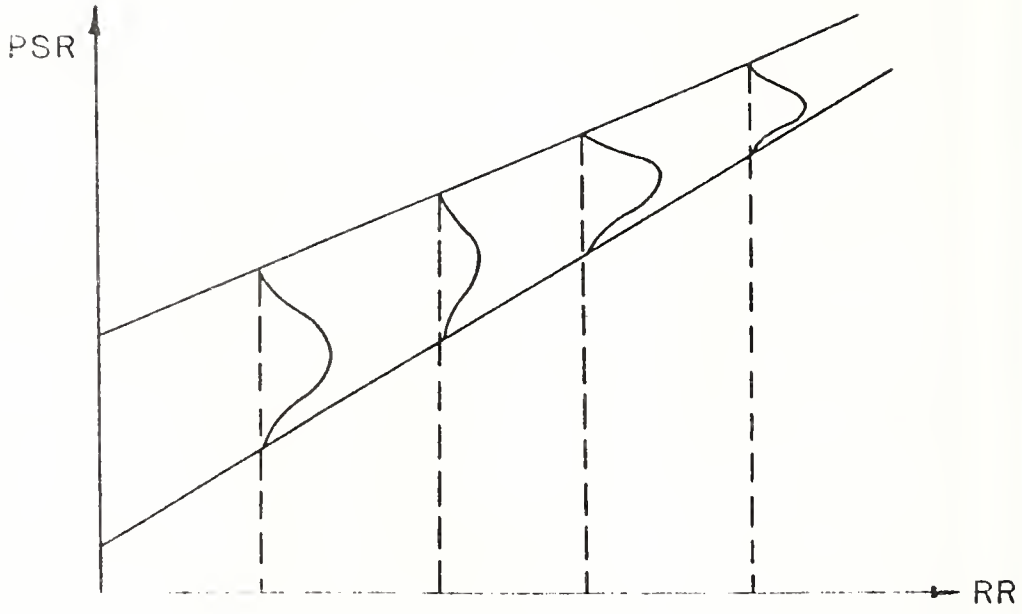


Figure 9. PSR - Roadmeter reading correlation

"vagueness" of the model is defined as:

$$J = \beta_1 + \ldots\ldots \beta_n \tag{64}$$

and the optimum solution of $\tilde{A}_i$ (γ_i, β_i) is the one which minimizes the vagueness of the model (J) subjected to the condition in equation (63).

The degree of fitting H can lie between 0 and 1.0 (Figure 10), with the best possible fit corresponding to a value of H of 1.0. Therefore any desired degree of fitting can be achieved by assigning a suitable value for H.

Once the fuzzy coefficients $\tilde{A}_i$, defined by γ_i and β_i are found using N data sets, the fuzzy set $\tilde{Y}$, corresponding to any other sets of data, x_i, can be obtained from equations (55) and (56).

In the special case of the PSR - RR correlation, equation (54) is simplified to (Figure 10) :

$$PSI = \tilde{A}_1 + \tilde{A}_2 \ (RR) \tag{65}$$

$\tilde{A}_1$ and $\tilde{A}_2$ are assumed to be π curves defined by the parameters (γ_1, β_1) and (γ_2, β_2), respectively. PSI is the fuzzy pavement serviceability index.

According to Appendix C, for a certain Roadmeter input and known A_1 and A_2 represented by π curves, the PSI obtained from the model is also a fuzzy set represented by a π curve with

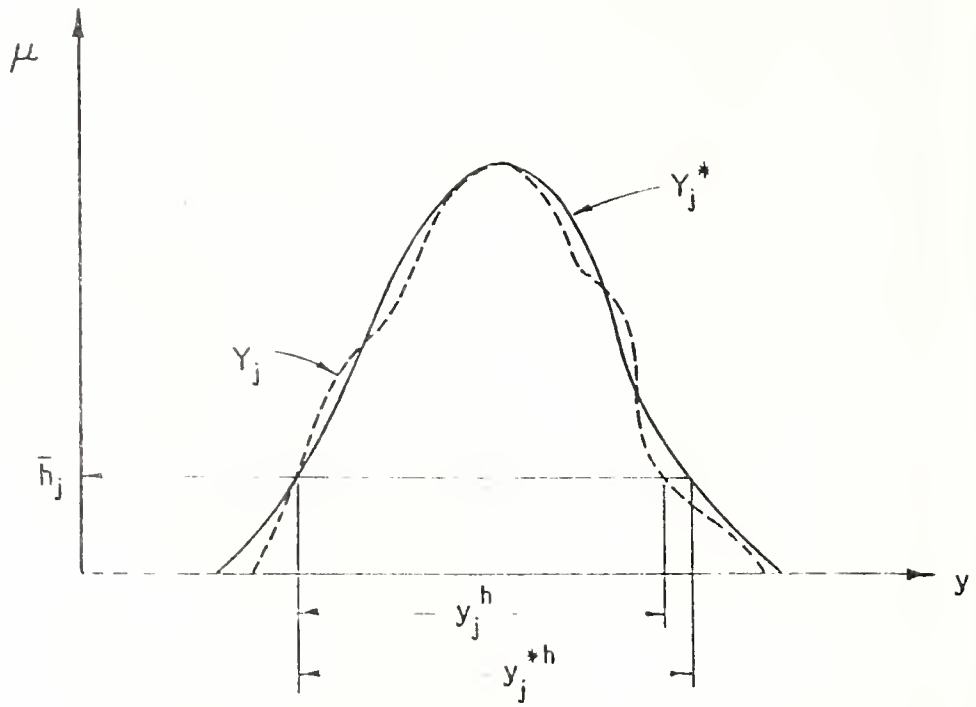


Figure 10. Degree of fitting

$$\gamma = \gamma_1 + \gamma_2 \text{ (RR)} \tag{66}$$

$$\text{and} \quad \beta = \beta_1 + \beta_2 \text{ (RR)} \tag{67}$$

Solution for $A_1(\gamma_1, \beta_1)$ and $A_2(\gamma_2, \beta_2)$ subject to the conditions in equations (63) and (64) was done using a computerized iterative procedure. With this solution, given the Roadmeter reading, the PSI of any pavement section can be found from equations (66) and (67) as a π curve.

As an example, the dark lines in Figures 11.a, 11.b and 11.c show the fuzzy PSR sets for three pavement sections, with Roadmeter readings of 500, 1000 and 1500 respectively.

By fitting a model PSI = $\tilde{A}_1 + \tilde{A}_2$ (RR) to these data and assuming π curves for A_1 and A_2 for a degree of fitness of 0.1, the following results were produced :

$$\tilde{A}_1 \equiv \pi (3.6, 0.98)$$

$$\tilde{A}_2 \equiv \pi (-0.0008, 0.00036)$$

With these data it was not possible to achieve a higher degree of fitness. Therefore it is seen that the degree of fitness (H) is not arbitrary and is often determined by the problem formation itself.

The dotted lines in the same figure represent the fitted curves in each case. For example from these results the extension principle yields the following fuzzy PSI set for a section

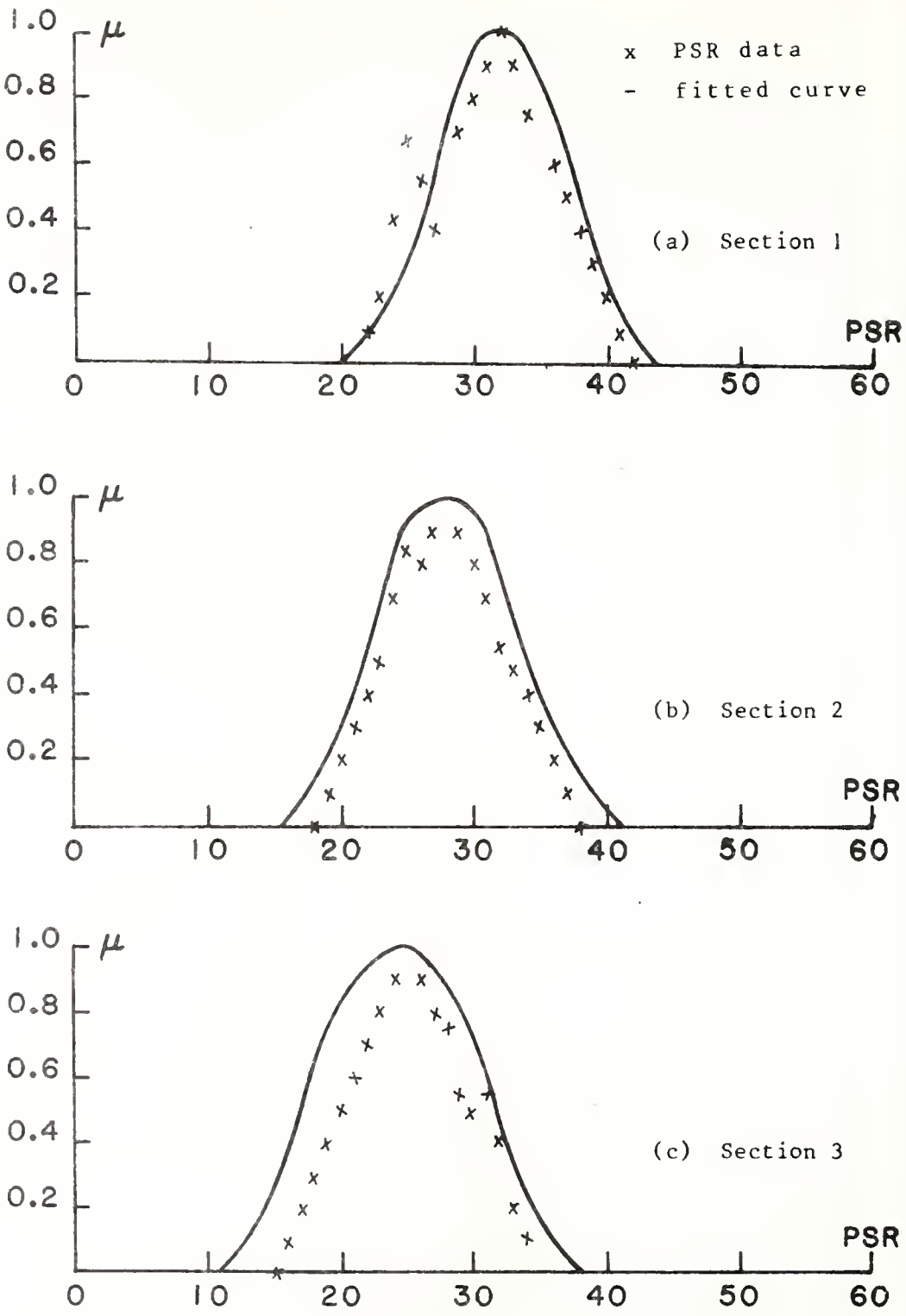


Figure 11. Results of Fuzzy regression

with a Roadmeter reading of 1250 :

$$PSI = \pi (2.6, 1.43)$$

In this chapter, a procedure was created for forming a fuzzy PSR model. Two alternative methods of obtaining the fuzzy PSI of a pavement section, through correlations with Roadmeter readings, are also outlined. However, this is only the first stage of the pavement management program. Later, it will be shown how this PSI, in its fuzzy form, becomes a tool to determine maintenance needs of the highway network.

CHAPTER 4 : PAVEMENT EVALUATION 2 : SKID, DEFLECTION AND DISTRESS

Skid-tester and Dynaflect variability

In Indiana, the skid-resistance and structural adequacy of a pavement section are evaluated using a trailer type Skid-tester and a Dynaflect, respectively. The Skid-tester measures the coefficient of friction of the pavement surface and converts it to a friction number, while the Dynaflect measures the deflection under a static load. Details of these instruments and their measuring systems are found in the literature (Mohan, 1978 and Metwali, 1981).

Several variables are known to affect these measurements and to introduce uncertainty into the recorded data. Several analytical techniques (Mohan,1978 and Metwali, 1981) have been suggested to handle this uncertainty based on statistical concepts. It is shown in this chapter that, although a part of this uncertainty is random in nature, system uncertainty also plays a major role. Several modifications to the present techniques are suggested herein based on the theory of fuzzy sets.

Inspection of the variability associated with skid-tester and dynaflect reveals many similarities. Four common sources of variation affect these two measurements. Further discussion will

be made in the light of these four sources.

Inability of repetition

It has been observed that repeated trials of the Dynaflect will result in different readings at the same location. This inability of repetition is a consequence of the lack of precision in the measuring system and is one type of system uncertainty, similar to the variability of the Roadmeter discussed in Chapter 3. Presently, the average of the readings of a specified number of tests is taken as the Dynaflect reading, considering this inherent imprecision as a random uncertainty. The specified number of test repetitions is decided upon by the null hypothesis procedure (Mohan, 1977).

It is proposed that, by introducing a "suitable" interval in place of a discrete reading, fuzzy sets theory can be used to handle this imprecision. Let us assume that the actual deflection, X, cannot be precisely measured and x is the average deflection obtained from repeated trials. This means that X has a possibility of taking a range of values around x. However, repeated measurements indicate an average of x suggesting that there is a high possibility of the actual reading lying very close to x, and lesser possibility of being away from it. Therefore, the actual reading can be considered as a fuzzy number represented by a membership function of L-L type with a maximum at x. The expert advice is only required to define the "spread" of this possibility distribution.

Then, the corresponding fuzzy reading (the fuzzified form of X) is given by :

$$X = [K]\ x \tag{68}$$

where K, the kernel of fuzzification (Chapter 3), can be used to define the required possibility distribution. K should satisfy all the conditions of an L-L type fuzzy set. Computation techniques to obtain X are described later in the chapter.

Variations along pavement sections

As the first step in ranking pavements according to their maintenance needs, the highway section is delineated into a number of contract sections each about 1 km in length. A fairly uniform sub-section 400 meters in length, within the contract section, is selected as a test section for measurements. Spot tests are conducted using Skid-tester and Dynaflect and the values obtained at a certain location represent the whole pavement section. Therefore, variations along a section clearly affect such a representation.

Currently, the average of a specified number of spot tests is treated as the friction number or the deflection of the section. The determination of such a number is also based on the null hypothesis. Assuming these sectional variations to be random in nature, it is best to account for them using the current statistical procedures. Hence, no modification is introduced herein for the sectional variations.

Uncertainty associated with conversion factors

In some cases it is neccessary to convert the results obtained under one set of conditions to another set of conditions, using correlation factors. For example, deflections are easily measured at the center of pavements but they must be converted to those at the edge, since edge deflections are more critical for design purposes. A marked seasonal variation in deflection values has also been observed (Metwali, 1981), the more critical ones being measured during the spring thaw period. Since it is more efficient to take measurements during the fall season, such values are converted to spring values, using correlation factors. Metwali (1981) proposed a regression analysis coupled with the analysis of variance method to obtain these factors. Imprecision enters these manipulations from at least three sources :

i. Correlation between quantities are often not exact, making it erroneous to use a precise factor. As an example, Metwali (1981) prepared figures correlating maximum spring deflections to those of summer and fall. Although he recommends that these curves be used to predict spring deflections, such crisp conversions are inaccurate and would cause errors in view of the scatter observed in the correlations.

ii. Mohan (1978) recommended that wherever possible, deflections should be measured at the edge location, i.e., 2 ft. from the

outer edge, except for pavements which have been widened. When old pavements have been widened, deflection should be taken at 2 ft. from the edge of the old pavements. This creates uncertainty in the definition of the "edge", especially for deteriorated pavements.

iii. Subjective judgment is involved in the selection of statistical tests. Mohan (1977) calculated the ratio of edge deflection to center deflection for all four kinds of pavements (asphalt, overlay, JRC, and CRC), based on the 80^{th} percentile values of the variance. He also recommended that these can be used to predict edge deflections from the center deflections for strengthening designs. The 80^{th} percentile was selected because it is required by statistical tests. However, the calculated factors can be different if a different percentile is used for this determination. It is therefore apparent that this "statistical subjectivity" also introduces imprecision to the factors.

In view of the above sources of uncertainty, the imprecise conversion factors can be represented as fuzzy sets supporting a range of values. Thus :

$$\Delta = f\ \delta \tag{69}$$

where

δ = measured deflection.

f = fuzzy correlation factor.

Δ = fuzzified deflection.

Since such a representation would consider all possible values of the factor, it is believed that it will result in a more meaningful procedure. Once these factors are fuzzified with the aid of experts, they can be used for regular operations, making use of the extension principle. This is illustrated later in the chapter.

Variability due to statistically insignificant factors

Various ambient conditions and other factors have been mentioned in the literature (Mohan,1978) to influence pavement evaluations, in a manner that cannot be statistically predicted. Therefore, these variations are not incorporated in the existing models. Effects of changes in ambient temperature, the rainfall and the vehicle speed on the friction measurements fall under this category.

An attempt is made herein to model these variations as imprecision in the measurements and to include them in the regular manipulations, using fuzzy tolerance analysis.

If x is a measured value, the fuzzified measurement is given by :

$$X = [K]\ x \tag{70}$$

where

$$K = K_1 \cdot K_2 \ \ldots\ldots\ K_n \tag{71}$$

is the composite kernel of fuzzification and K_i are the kernels

associated with each type of variation. Influence of gas tank level, driver characteristics and climatic changes are similar factors that affect the Roadmeter count. This problem was addressed in Chapter 3 using techniques similar to those of equations (70) and (71).

Fuzzification of friction number and deflection

Combination of kernels associated with different factors (equation (71)) was described as a means of reducing computer time and storage in Chapter 3. Kernels are of two types: those which take a discrete set of values, and others with a continuous support domain. Some mathematical techniques for operations with any types of kernels are discussed in the Appendix D, with three cases being discussed in detail : (1) combination of two discrete kernels; (2) combination of two continuous kernels; and, (3) combination of a discrete kernel and a continuous kernel.

These procedures can be extended for any number of fuzzy kernels by treating the composite kernel at any intermediate stage as a new kernel. The ultimate one, depending on whether it is discrete or continuous, can operate on the recorded reading according to cases (4) and (5) in the Appendix D. Discrete sets can be stored in arrays containing each individual membership, and a new, and usually larger, array is developed for the composite kernel. However, most of the present work involves continuous kernels, as pavement deflection and friction number are

continuous variables. For numerical purposes, continuous fuzzy sets can also be represented by arrays which store the memberships of a series of values separated by an appropriate interval. The accuracy of the result can be increased by making this interval as small as possible, and the manipulations can be carried out as in the case of discrete sets. The computer programs SKID and DEF have been developed to : (1) Incorporate the effects of temperature and rainfall on friction number (equations (70) and (71)); and, (2) Introduce tolerance intervals in the Dynaflect reading due to its irrepeatability. (equation (68)).

Two questionnaires (Nos. 3 and 4) were prepared to seek highway engineers' opinions on the extent of variation (as a percentage) of a measured friction number due to changes in vehicle speed and climatic conditions, and the tolerance interval (as a percentage) to be introduced into the deflection to account for Dynaflect irrepeatability. Kernels were formed based on π curves, with the variation or tolerance on either side of the mean value assigned to β in equation (29). These continuous kernels are then discretized using an interval of discretization selected by the users of DEF and SKID computer programs. Listings of these programs are given in the Appendix H.

Use of inexact factors

It was shown earlier that fuzzy sets can be used to express imprecise factors used for conversions. Important mathematical results useful for dealing with such fuzzy factors are presented

below for discrete factors. Continuous factors can always be discretized for computer manipulations.

Multiplication of a crisp number by a fuzzy factor (Figure 12) :

The fuzzy factor represented by :

$$f = \underset{i}{U}\ \mu_i/y_i \qquad y^* - \theta \leq y_i \leq y^* + \theta \tag{72}$$

when multiplied by the crisp number δ yields :

$$F = f \cdot \delta$$

This can be manipulated by applying the extension principle,

$$F = \underset{i}{U}\ \mu_i/y_i \cdot \delta$$

If $x_i = y_i \cdot \delta$

the final result will be as follows :

$$F = \underset{i}{U}\ \mu_i/x_i \qquad \delta \cdot(y^* - \theta) \leq x_i \leq \delta\cdot(y^* + \theta) \tag{73}$$

Similarly, if f is continuous (Figure 13), the final result is :

$$F = \int_{\theta} \mu/x \qquad \delta \cdot(y^* - \theta) \leq x \leq \delta\cdot (y^* + \theta) \tag{74}$$

Multiplication of a fuzzy number by a discrete fuzzy factor (Figure 14) :

This multiplication is useful when converting a fuzzified reading corresponding to a given set of conditions into a reading for a different set of conditions, using a fuzzy factor. Let the

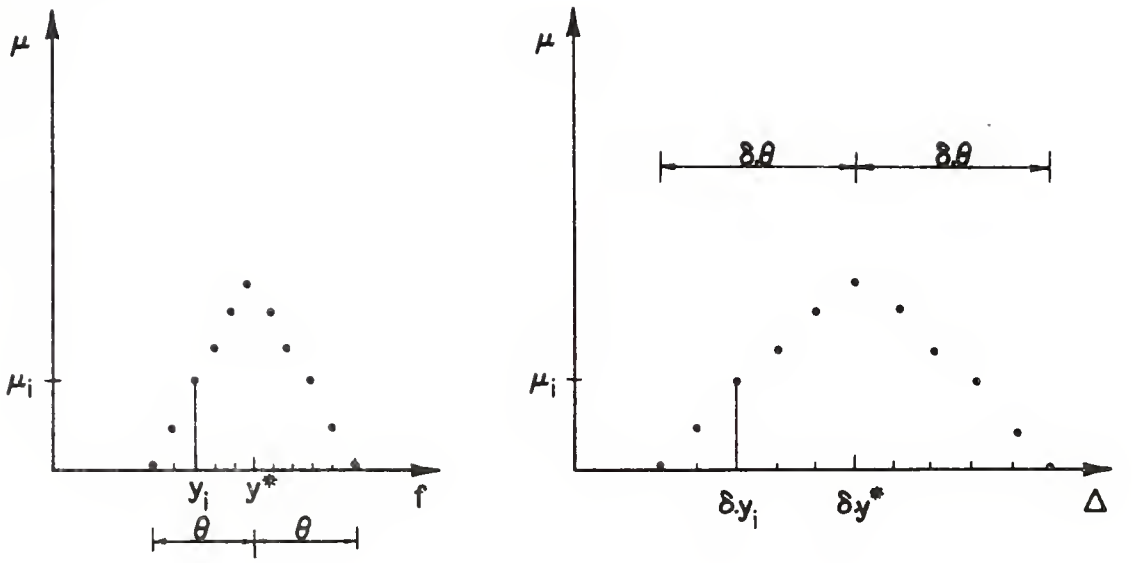


Figure 12. Discrete fuzzy factor

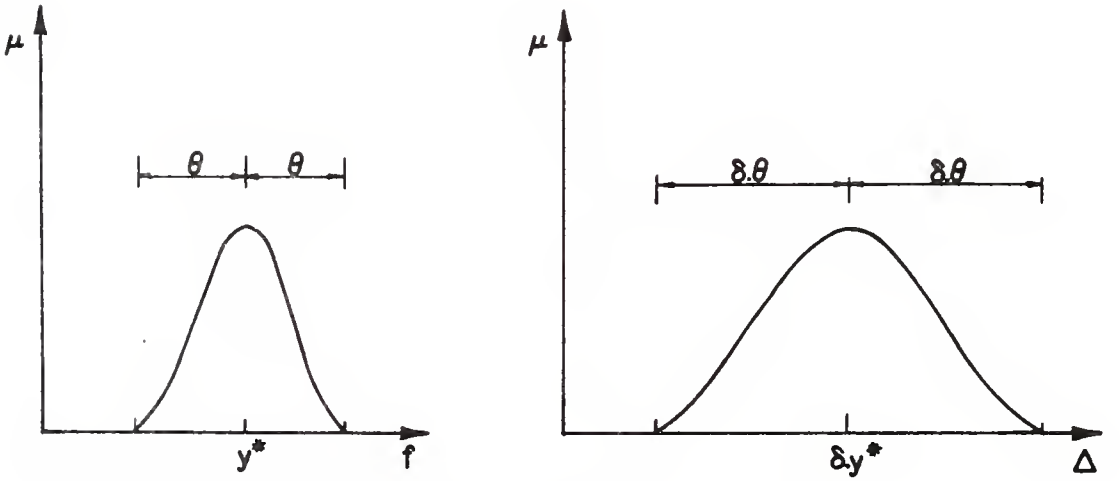
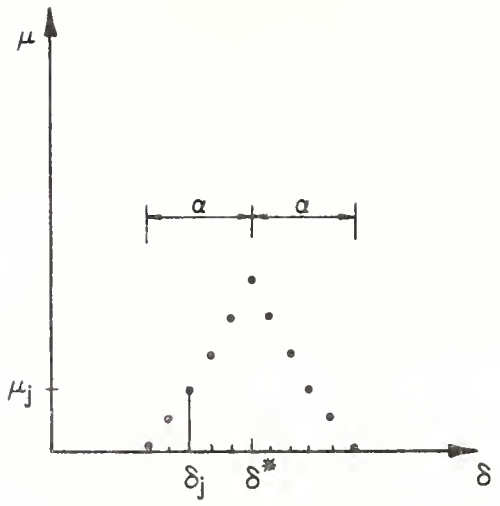
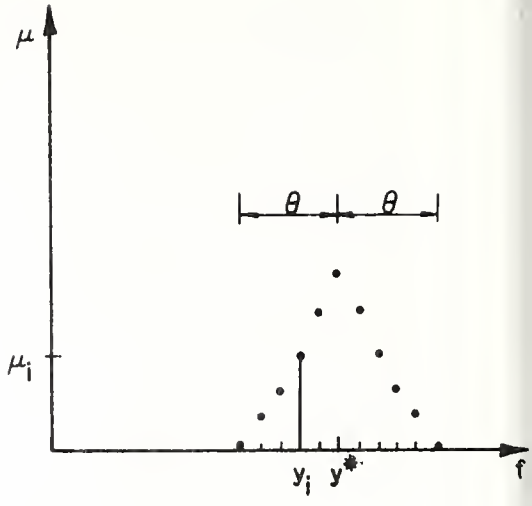


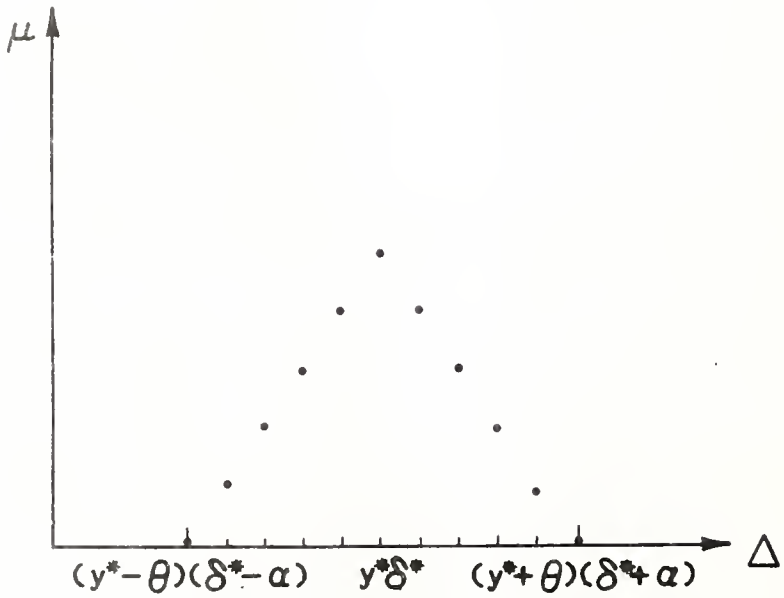
Figure 13. Continuous fuzzy factor



a) fuzzy number



b) fuzzy factor



c) product

Figure 14. Multiplication with a fuzzy factor

fuzzy factor be given by equation (72) and the fuzzy number by :

$$\delta = \underset{j}{U}\ \mu_j/\delta_j \qquad \delta^* - \alpha \leq \delta_j \leq \delta^* + \alpha \tag{75}$$

Using,

$$F = f.\ \delta$$

and by substituting for f and δ gives :

$$F = \left(\underset{i}{U}\ \mu_i/y_i \right) \left(\underset{j}{U}\ \mu_j/\delta_j \right)$$

This can be manipulated by applying the extension principle :

$$F = \underset{i}{U}\ \underset{j}{U}\ \mu_i \wedge \mu_j/y_i . \delta_j$$

The final result can also be written as :

$$F = \underset{i}{U}\ \underset{j}{U}\ \mu/x_k \tag{76}$$

where,

$$x_k = y_i \delta_j$$

$$\mu = \mu_i \wedge \mu_j$$

and $(y^* - \theta)(\delta^* - \alpha) \leq x_k \leq (y^* + \theta)(\delta^* + \alpha)$

Other cases of operations involving fuzzy factors are described in the Appendix D.

The method of obtaining the extents of uncertainty, to form the kernels and fuzzy factors, will now be discussed. For this,

the following information was sought from the experts (see Questionnaire No. 4, Appendix F) :

i. a. Factors used for each type of pavement (asphalt, overlay, CRC and JRC) for the conversion of fall and summer deflections to spring deflections.

   b. Possible tolerances of these factors (as percentages of the factors itself) due to all sources of imprecision discussed in cases (i),(ii), and (iii) of the previous section on uncertainty in correlation factors.

ii. a. Factors used for each type of pavement for the conversion of deflections to those at the edge.

   b. Possible tolerances of those factors (as a percentage of the factor) due to the same sources of imprecision as in (i)b.

Fuzzy factors were represented by π type fuzzy sets with the mean values assigned to γ and tolerances on either side of the mean assigned to β in equations (29a) and (29b). The next step was to discretize these fuzzy sets to enable them to be easily handled by the computer program DEF. One application of the use of fuzzy factors is illustrated in the following example.

A design application

The Indiana Department of Highways uses the Asphalt Institute's curves to obtain overlay design thicknesses for

asphalt pavements, using Dynaflect readings. The Asphalt Institutes curves are reproduced in Figure 15 where the representative rebound deflection is to be measured by the Benkelman Beam (BB), and the parameter DTN abbreviates the design traffic number.

The relationship between the dynaflect maximum deflection(DMD) and the Benklemen Beam(BB) deflection is given by (Metwali, 1981) :

$$BB = 20.\ DMD \tag{77}$$

In current practice, variation in Dynaflect reading is recognized and the representative value of the deflection is selected as the mean deflection, δ, plus two standard deviations, σ. Assuming a normal distribution for δ, this value would encompass 98% of all possible values. On the other hand, a fuzzy deflection set obtained by fuzzification techniques encompasses all possible deflection values.

The following example illustrates the potential use of Asphalt Institutes curves using fuzzy sets techniques. Assume that the deflection reading obtained by a Dynaflect at the edge of an asphalt pavement is $\delta = 3$ mils during the fall season. If the discretized kernel that accounts for the imprecision of the reading due to the irrepeatability is :

$$K = 0.8/(y - 0.1) + 1.0/y + 0.8/(y + 0.1)$$

and if the ratio of spring deflection to fall deflection, for an asphalt pavement, is given by the factor:

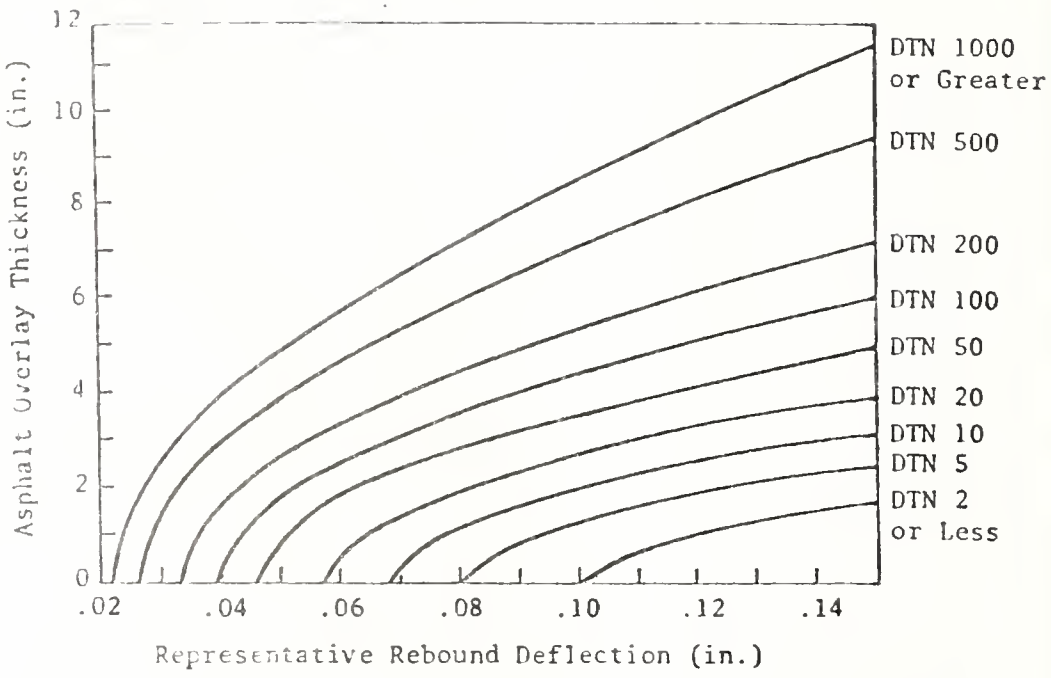


Figure 15. Required overlay thickness as a function of deflection (from Asphalt Institute)

$f = 0.7/1.4 + 1.0/1.5 + 0.7/1.6$

then, by using the fuzzification techniques discussed previously, operating K on δ gives :

$$\Delta_{fall} = 0.8/2.9 + 1.0/3.0 + 0.8/3.1$$

Similarly, using the fuzzy conversion factor,f, the corresponding spring deflection (in mils) is obtained as:

$$\Delta_{spring} = f. \Delta_{fall}$$
$$= (0.7/1.4 + 1.0/1.5 + 0.7/1.6)(0.8/2.9 + 1.0/3.0 + 0.8/3.1)$$
$$= 0.7/4.1 + 0.7/4.2 + 0.8/4.4 + 1.0/4.5$$
$$+ 0.8/4.7 + 0.7/4.8 + 0.7/5.0$$

Note that these results show how the number of membership values to be stored increases significantly after each operation.

The corresponding BB deflection from equation (77) would be (in inches) :

$$\Delta_{BB} = 0.7/0.08 + 0.7/0.085 + 1.0/0.09 + 0.8/0.095 + 0.7/0.1$$

If the design DTN is 200, the fuzzified overlay thickness (from Figure 15) can be obtained as :

$$T = 0.7/0.41 + 0.7/0.46 + 1.0/0.5 + 0.8/0.52 + 0.7/0.53$$

It appears reasonable that the highest value with a significant membership will be a safe choice. Thus, a thickness of 0.53 inches, could be used in this design. It should be noted that all possible deflection values were considered in this method. On the other hand, if the conventional method is used assuming a

standard deviation of 0.1 mils, a design thickness of 0.52 inches is obtained. It should also be noted that the latter approach uses a deflection greater than the mean by two standard deviations, which approximately corresponds to the 98^{th} percentile of the maximum possible deflection.

The fuzzy sets method is also advantageous over the existing method in that it can easily account for uncertainties in the DTN such as a DTN of "about 200".

Pavement distress surveys

Suitability of a highway pavement as a transport facility is defined by its rideability, structural adequacy, skid-resistance and distress manifestation. Evaluation of a pavement involves consideration of all these properties. In the preceeding sections, it has been attempted to form fuzzy sets methodologies to overcome the system uncertainty present in the evaluation of the first three properties. Fuzzy sets mathematics can also aid in the distress evaluation of a highway pavement.

Distress is any indication of poor or unfavourable pavement performance or signs of impending failure. All such signs can be grouped into three main classes of distortion, disintegration and fracture. Distress is depicted in different pavements in the following forms :

Flexible pavements:

a. Transverse cracks

b. Longitudinal cracks

c. Alligator cracks

d. Block cracking

e. Rutting

f. Shoving

g. Patching

h. Excessive asphalt

i. Pumping and water bleeding

j. Corrugations

k. Ravelling

l. Polished aggregate

Rigid pavements:

a. Transverse cracks

b. Longitudinal cracks

c. Patching

d. Pumping and water bleeding

e. D-cracking

f. Crack spalling

g. Faulted transverse joints

h. Longitudinal joint seperation

i. Pavement break-up

The evaluation of the distress condition of pavements is an essential part of the pavement evaluation process and the selection of remedial measures. A literature survey (TRB,1981 and

Pavement Management,1983) reveals that most states in the USA conduct condition surveys on a routine basis, but without any objective measurements.

Pavement condition rating

The Indiana Department of Highways is presently developing a procedure for condition (or distress) surveys of pavements. This procedure will be similar to the survey techniques used by other states such as Arkansas, Florida, Idaho, Nevada, Ohio, Washington, Arizona and California. The common practices of some of these states are outlined below.

Surveys are conducted by two crews, each with two persons, usually from the planning division of the department of highways. Training sessions are held periodically to instruct new survey personnel and to refresh others. The crew drives to a designated mile-post, exits the automobile and walks the length of the sample section. This length is often selected as 100 feet for flexible pavements and 300 feet for rigid ones. Different types of pavement distresses are then carefully examined depending on the pavement type.

In all pavement sections, each defect type is rated according to its severity and extent. Distress survey instruction sheets are used for this purpose. Then, the distress rating is determined by deducting the ratings for for all defect types from an initial value of 100 (TRB, 1983). This rating is known as the

pavement condition rating (PCR). Thus, a pavement with no defects will obtain a maximum score of 100. In some states such as Idaho, the routine distress evaluation consists of recording the extent of cracking only. On the other hand, several states include distress measurements such as cracking, patching and rutting in the equations relating the roadmeter reading to the PSI. For the State of Indiana, Mohan(1978) developed such correlations, but they are not in use at present. Instead, the Indiana Department of Highways (IDOH) is using a simplified PSR-RR correlation which does not include distress.

The Ohio Department of Transportation designates certain types of distress, depending on the pavement type, as reflecting structural problems (Pavement Management, 1983). Rating them enables a total number of "structural deduct" points to be calculated. If a specified structural deduct number is reached, depending on the class of the roadway, a Dynaflect test is scheduled to measure the section's structural adequacy.

Levels of importance are attached to each class of defects. These levels differ with the type of pavement. Each distress type is to be rated on a scale (e.g. 0-5) which represents the relative importance of that distress type. Such scales have been established based on experience of highway engineers and on the results of the other states experience (TRB, 1981). IDOH has prepared its own instructions for rating flexible and rigid pavements separately. These instructions are found in the Appendix E. Recently it has been decided by the IDOH to change the condition

survey procedure and rate fewer cracks in order to make the procedure more efficient. The proposed procedure is shown in Table E1.

Subjectivity in rating procedure

Subjectivity enters the rating procedure because the rating is determined by approximate measurement or estimation of the distress extent and severity according to the instructions. Although some states insist on photologging (TRB,1981), the condition of the selected pavement is only visually compared to standard photographs in determining the rating. This procedure also introduces human based uncertainty into the rating procedure.

In general, three sources of uncertainty can be identified in the distress rating procedure : (1) Determination of extent of the defect to be rated ; (2) Determination of severity of the defect to be rated ; and (3) Variability of the distress along the section. Most of the measurements that determine the extent of a defect are imprecise. This particularly is the case with approximate area estimations. This invariably introduces imprecision and vagueness to the estimation of the extent of the defect. Moreover, raters are instructed to assign higher ratings to severe defects. Therefore, a fuzzy representation of the distress rating is appropriate to model the human based uncertainty in the estimation of both extent and severity of the defects, and to manipulate the linguistic variables. Two alternate methods

are proposed herein to accomplish this; fuzzy PCR and damage assessment methods, respectively.

Measurements carried out at designated mile-posts on the pavement sections determine the distress rating for the entire section. Hence, variability of distress manifestation along a pavement section introduces additional uncertainty into the distress rating. Throughout this work, the variability of a certain pavement property along the test section has been identified as a random uncertainty; accordingly, it is left to be treated by statistical theories.

Fuzzy PCR concept

This is an extension of the conventional pavement condition rating method. First, the subjective rating assigned for a particular distress type (cracking, patching or rutting etc.) on a certain pavement section is fuzzified. This is done by means of two kernels representing the imprecision introduced by the extent and the severity of the distress. The fuzzy rating which can be visualized as having a support region on either side of the assigned rating, can then be obtained from equations (70) and (71). Different kernels of fuzzification can be defined for different distress types using expert knowledge, and kernels associated with extent and severity will be different even for the same distress type. Questionnaire No.5 (Appendix F) was prepared to obtain the judgment of highway experts on this fuzzification technique. This questionnaire was prepared in the same manner as

the questionnaires discussed previously, which also sought the possible extents of vagueness. At this stage the reader will realize that the best sets of experts are the distress survey crew members themselves.

Finally, the fuzzified ratings for all distress types are added and the result is substracted from an initial value of 100, using fuzzy addition and substraction. The extension principle (Chapter 2) becomes useful in these operations.

As an example, let us assume that a crew has assigned a rating of 5.0 to a certain flexible pavement section, for alligator cracking, based on the instruction sheet in the Appendix E :

$$\theta_{al.crack} = 5.0 \tag{80}$$

Further, assume that the fuzzification kernels associated with extent and severity of alligator cracking are given by $\pi (x , 0.1)$ and $\pi (x , 0.2)$, respectively. Thus, with symmetry observed around x (the kernels are discretized for convenience) :

$$K_{extent} = 0.125/(x-0.075) + 0.5/(x-0.05) + 0.875/(x-0.025) + 1.0/x \tag{78}$$

$$K_{severity} = 0.05/(x-0.175) + 0.125/(x-0.15) + 0.28/(x-0.125) + 0.50/(x-0.1) + 0.72/(x-0.075) + 0.875/(x-0.05) + 0.96/(x-0.025) + 1.0/x \tag{79}$$

After successive fuzzification of equation (80) by equations (78) and (79) :

$$\theta_{a.c} = 0.007/4.75 + 0.017/4.775 + 0.065/4.8 \qquad (81)$$

$$+ 0.14/4.825 + 0.25/4.85 + 0.44/4.875 + 0.63/4.9$$

$$+ 0.77/4.925 + 0.875/4.95 + 0.96/4.975 + 1.0/5.0$$

This fuzzy rating can be interpreted as having a support range spreading from 4.75 to 5.25 (symmetry with respect to 5.0), with the value of 5.0 taking the highest membership.

This procedure can be repeated to obtain θ_i for all the distress types (i=1.....n) on the pavement section. Figure 16 shows a plot of several such fuzzy sets on a scale of 0-10. The current method of obtaining the PCR can now be extended by using the extension principle:

$$\mu_{PCR}(y) = \sup_{y=100-\Sigma x_j} \min_i [\mu_{\theta_i}(x_j)] \qquad (82)$$

where $0 \leq x_j \leq 10$ or $0 \leq x_j \leq 5$ depending on the type of distress, and i is the distress type. y is any value supporting the PCR of the section. It follows that $0 \leq y \leq 100$ and therefore, the PCR is a fuzzy set on the scale of 0-100 (Figure 17).

This is called the "fuzzy PCR" for the pavement section. A range of values in a scale of 0-100 support the PCR of the section. This range and the associated membership values depict the

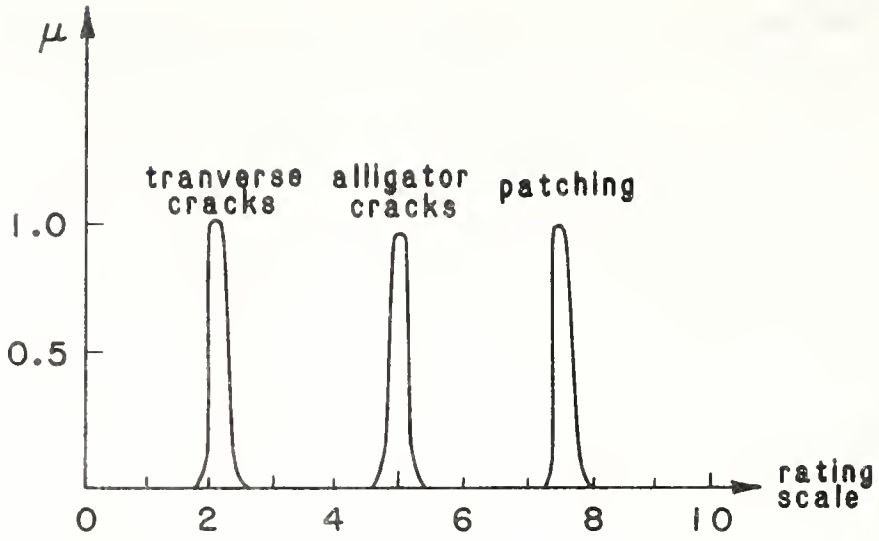


Figure 16. Fuzzified ratings for different distresses

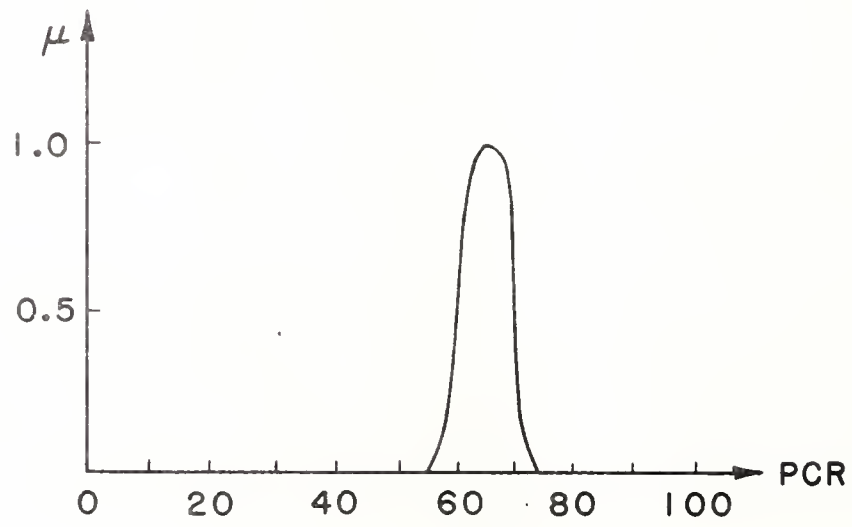


Figure 17. Fuzzy pavement condition rating

uncertainty associated with the judgement of the rating crew in assigning a single number to represent the distress condition of the section.

Damage assessment method

Distress manifestation can also be viewed in the context of damage assessment. Ishizuka et.al.(1981) proposed a method of applying expert knowledge in assessing damage in existing structures. A similar method is developed here to apply expert knowledge in pavement distress surveys. This new methodology essentially deviates from the existing distress surveys but has the potential of meeting the needs for subjective evaluation, as required by such a survey.

In this method the crew members are considered as damage assessment inspectors and the various forms of distress are analogous to the damage in various components of a structure. As a first step, a linguistic damage scale (Figure 18) is defined. The different damage grades are identified by the linguistic variables "slight", "moderate" etc. The raters can then be asked for their belief that each distress type on a given section belongs to each damage grade. If these beliefs are expressed in a scale of 0.0-1.0, they can be treated as memberships of each distress type on the corresponding fuzzy damage grade. Subjective judgement aided by measurements, careful examination and experience will determine these membership values.

Figure 18. Damage scale

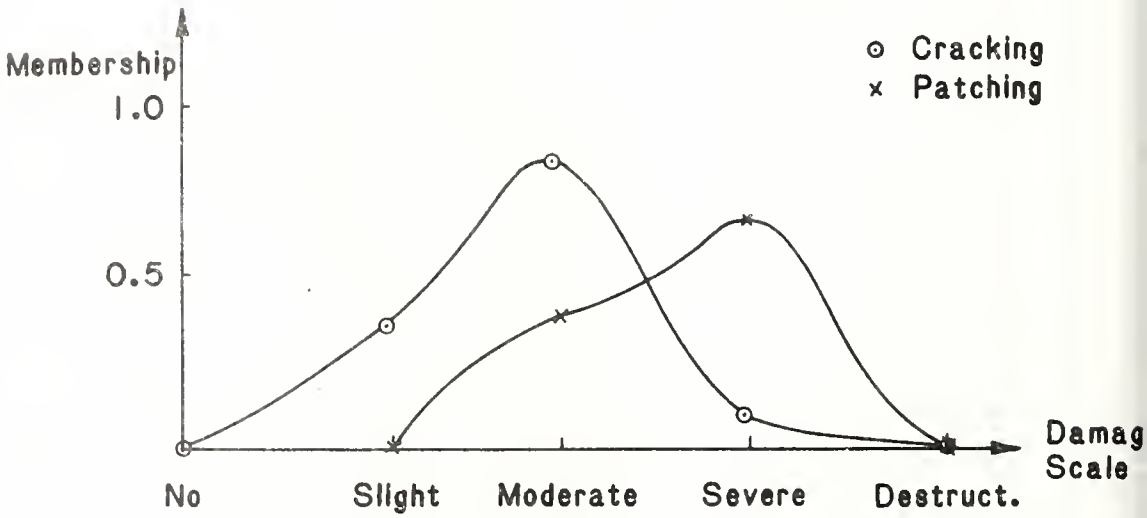


Figure 19. Membership of distress types in damage grades

Once the whole pavement section has been examined, fuzzy sets representing each defect are automatically obtained on the damage scale (Figure 19). As an example, suppose that the rating crew assigned the following memberships, on inspecting cracks and patches on a flexible pavement section :

	cracks	patches
Slight distress	0.2	0.0
Moderate distress	0.6	0.5
Severe distress	0.2	0.6

This evaluation results in the following fuzzy set:

$$\theta_{cracking} = 0.2/Sl + 0.6/M + 0.2/Se$$

$$\theta_{patches} = 0.5/M + 0.6/Se$$

where Sl,M and Se stand for the linguisic damage grades, slight, moderate, and severe.

As the global damage of a structure is the "sum" of damages to the individual components, the overall distress rating of a pavement section is the "sum" of the the individual distress ratings. Following this idea, fuzzy sets for each distress types can only be aggregated using either the union operator, or algebraic summation or fuzzy addition. The fuzzy addition, used in the fuzzy PCR method, cannot be performed on a linguistic damage scale. It is proposed that the union operation be performed in preference to algebraic summation in order to incorporate relative weights of each distress type (w_i), using :

$$\theta_T = \underset{i}{U}\, w_i\ \theta_i \tag{83}$$

where

$$\sum_i w_i = 1 \tag{84}$$

The same method and similar equations were previously used for the aggregation of subgroup opinions (equations (32)) in the PSR formulation.

The simplest method to obtain these weights is to normalize the maximum values of the corresponding scales. If cracking and patching are rated on scales of 0–5 and 0–10 respectively, it can be assumed that 5:10 is a reasonable representation of the relative importance of the two distress types. These scales (Appendix E), have been designed using highway engineers' experience and therefore would certainly be reasonable representations of the relative importance.

For the cases of cracks and patches, the following weights are obtained : (equation 84) :

$$w_{cracks} = 0.33$$

$$w_{patches} = 0.67$$

Then, the overall damage due to cracking and patching is given by (equation (83)):

$$\theta_T = 0.17/S1 + 0.83/M + 1.0/Se$$

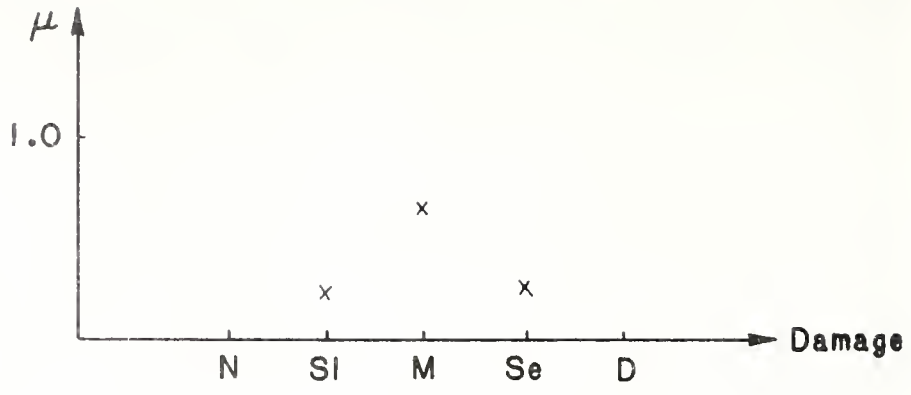
Figure 20 shows component and resultant fuzzy sets. Note that the previous component fuzzy sets are normalized before the aggregation.

In the fuzzy PCR method, no weights need to be attached to fuzzy ratings for different distresses, as the scales which depict the relative importance of each distress type are cumulatively added in the fuzzy addition. In other words, since different scales are used for different distress, the relative importance of each type is automatically included in fuzzy addition through these different scales. On the other hand, if the second method is used it is not neccessary to seek the opinion of experts on the rating subjectivity. This is already manifested in the linguistic damage scales. A disadvantage of the latter method is that, in its present linguistic form, it cannot fit into the proposed global pavement evaluation model.

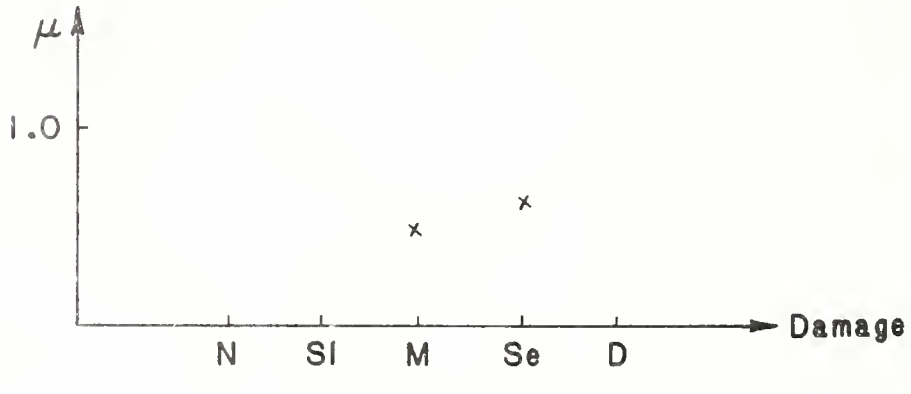
Expert knowledge

The need for expert knowledge and experts opinion has been mentioned consistently in the development of the proposed methodology. An attempt to form a rationale for selecting a panel of experts and extracting expert knowledge is made here.

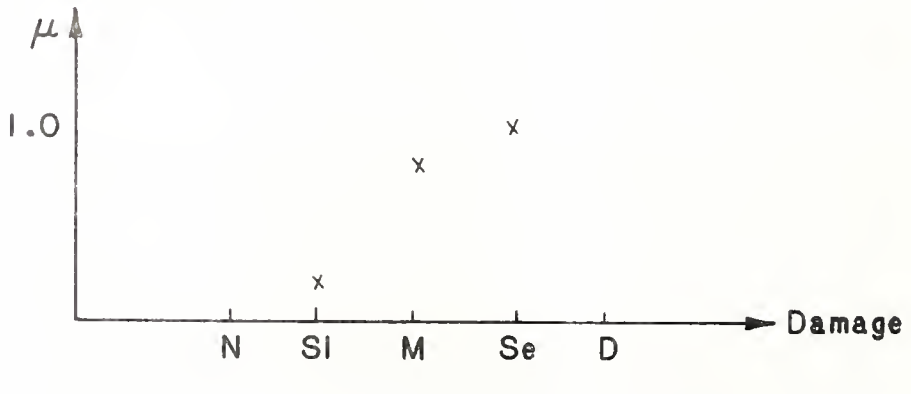
A literature survey reveals that very little has been done in the fuzzy sets arena towards developing a method of selecting experts. Work involving selection of experts is published in the area of decision making using subjective probability assessments.



a) Crack rating



b) Patch rating



c) Resultant rating

Figure 20. Crack, patch and resultant ratings

These techniques seem to be appropriate for the present work, since many similarities exist between membership functions and subjective probability assessments, despite their difference in concept.

In some cases, historical data records provide an alternative to the response of experts. Variations of the roadmeter reading with gas tank level and of the friction number with climatic conditions fall into this category. However, questions pertaining to the perceptiveness of raters, extents of variation associated with the ratings and the like are best answered by knowledgeable technical personnel working extensively in the particular areas.

To avoid any personnel bias, mistakes or lack of knowledge of a given individual, each component of the knowledge base must be produced by a team of experts. This property is known as group elicitation (Hogarth, 1975).

Because of the versatile aspects of the knowledge base, its establishment would require several teams of experts, each of which excels in one aspect. Each team is assigned a "module" within the framework of its expertise. In the present case, different modules must be set up for different speciality areas such as rideability, friction, etc. This is the modularity property of a panel (Hogarth, 1975).

Two important things must be remembered when building a knowledge base using experts. First, the human mind is not very

efficient in aggregating the significance of a number of features simultaneously. Thus, complex problems must be subdivided into simpler questions which are easy to be answered. This is known as the problem reduction method. For example, when dealing with Roadmeter variability, variations due to gas tank level, driver characteristics and climate were considered case by case in the questionnaires (No. 2) with only one factor varied at a time. Also, simple numerical examples were provided in the questionnaires wherever possible. Second, it is very likely that a high quality knowledge base will not emerge after the first round of sessions with experts. Therefore, the possibility of update or "tune-up" has to be provided. In view of this, the computer programs ROAD, DEF, and SKID are treating data to be obtained from experts as input parameters.

These pre-requisites for selection of experts indicate that the Delphi techniques (Dalkey, 1969) can be suitable to achieve the desired expert knowledge. The Delphi method was originally developed by the Rand corperation in the early 1950s for the Air Force and was intended to improve the validity of decisions reached by group consensus. Selection of the panel of experts is one of the most important areas in Delphi design. Here, an expert is defined as a person with a high degree of knowledge in the particular area, which may be attained through original research, field experience or other means. In view of this common area of interest, it is suggested that the guidelines for expert panel selection in the Delphi method be used in the selection of

experts for applications of fuzzy sets.

One basic hindrance to any expert selection procedure is that if the panel is too diverse it may be necessary to place experts into subgroups and weight the response of each subgroup according to its expertise on the question concerned. This can be overcome in two ways. Practical applications of Delphi techniques are cited (Corotis et al,1981), where only engineers who were held in high esteem by their peers were selected in order to obtain the most valuable opinions and thus avoid having to weight the responses. It is also possible to first conduct a policy Delphi to establish a common base of knowledge and consistent definitions to be used subsequently. This step would certainly reduce disparities in expertise within the panel.

In forming the expert panel for the purpose of pavement evaluation, highway engineers from the Indiana Department of Highways and the Federal Highway Administration were consulted through arranged meetings at Purdue university. Each of the participating engineers were asked to nominate a number of engineers for each module such as rideability, friction etc. corresponding to each area concerned. Prototype questionnaires distributed at the meetings aided them in this venture. The next step involved selecting the engineers whose names have been proposed by many of their peers and persuading their participation in the panel through telephone conversation.

One important feature of a normal Delphi technique is the continuous feed-back of responses until convergence to a group

consensus is achieved. However in applying fuzzy sets techniques it is the expert opinion that is sought rather than a group consensus. Thus it was decided not to enter the feed-back stage unless responses of the individual experts are far too diverse.

One major problem encountered in expert selection was the unavailability of an adequate number of highway engineers in Indiana, respected by their peers as knowledgeable in each area dealt in this research. In contrast to expectations of about 20 experts in each module, the maximum number of responses obtained for each questionnaire ranged from 5 to 15.

CHAPTER 5 : DECISION MAKING : PRIORITY ASSIGNMENT

Decision sequence

The pavement management system (PMS) requires collection of different types of data. The two main types are performance data and traffic data. Performance of a pavement section is determined by four properties : rideability, skid-resistance, structural adequacy and distress manifestation. Traffic data, which express the traffic concentration upon a pavement, are usually obtained from traffic surveys. In addition, data concerning pavement geometry and hazard locations are also included in the PMS. Presently, in Indiana, as in many other states, some or all of the pavement data are collected at the highway network level. In this chapter it is shown how these pavement data can be utilized in conjunction with traffic data to assign maintenance priorities for all pavements.

Budgetary, time, and labor restrictions preclude making all four performance measurements at every pavement section. However, all sections are first scanned with the roadmeter and the measured roughness is converted to the PSI scale. The PSI is then compared with the acceptable serviceability rating to determine the next course of action. Dynaflect testing and distress surveys are performed on the unacceptably rough pavements while the

remaining pavements are subjected to skid-tests. Measured friction numbers are also compared with acceptable friction levels for maintenance purposes. It will be shown that acceptable serviceability and friction do not have precise values on their respective scales. The proposed technique can make allowance for this imprecision.

As pointed out in earlier sections, the presence of human based and system uncertainties in the performance data suggests a more meaningful representation of these data by using fuzzy sets. As a first stage of decision making, procedures are described in this chapter to enable the screening of sections by comparison of fuzzy pavement data with fuzzy acceptability levels. Traffic data, on the other hand, also inherit uncertainty. Since this uncertainty is mainly random in nature, it is reasonable to represent traffic data using a crisp Average Daily Traffic count (ADT).

The next task is to combine all the types of data and create maintenance priorities. At this decision stage, the subjective judgments of the decision makers come into play, even more so when the number of performance data is large. The theory of fuzzy sets can assemble the subjective opinions of the decision makers. Starting from responses to simple queries, use is made of this theory to build an expert knowledge base with regard to assignment of priorities. Fuzzy sets techniques can then be utilized to determine the relative maintenance priority of each pavement section by the interaction of fuzzy or random performance data with

the expert knowledge base containing priority utilities.

Acceptable serviceability index

The serviceability level above which a pavement will adequately serve the users depends on whether it is a primary or a secondary facility. For a given type of facility, this level depends on individual judgment, since different people view pavement needs differently. This was clearly shown in the responses to the questionnaire No. 2 (Appendix F), where a selected panel of highway engineers was asked to indicate acceptable serviceability indices (ASI) for the two types of facilities. Many experts preferred to indicate a small interval for the ASI, being unable to decide on a unique value. Combination of these values produces a range of ASI with varying degrees of beliefs, making it an ill-defined quantity. Thus, if each of these ASI values are plotted against the number of experts favoring that ASI, a figure similar to Figure 21 is obtained. An acceptable serviceability range was found to be more useful than the ASI, for reasons to be described. Such a range will include all the PSI values which are acceptable.

A method resembling "exemplification" (Zadeh, 1972) can be used to form the memberships of the acceptable serviceability range. Due to different opinions, the number of experts that considers a given PSI value as acceptable, differs from value to value. This provides a convenient way of forming the desired membership function. The strength of belief of any value x being

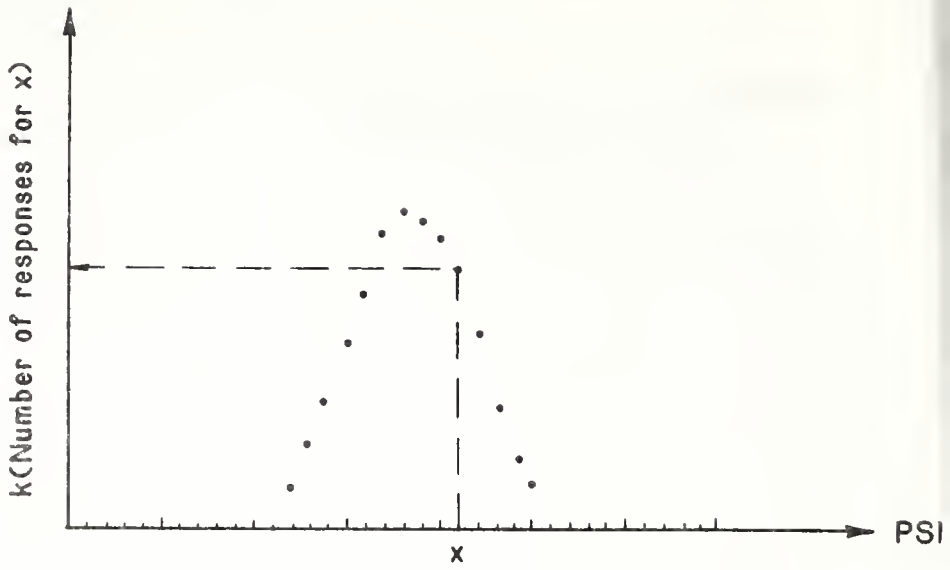


Figure 21. Acceptable serviceability rating

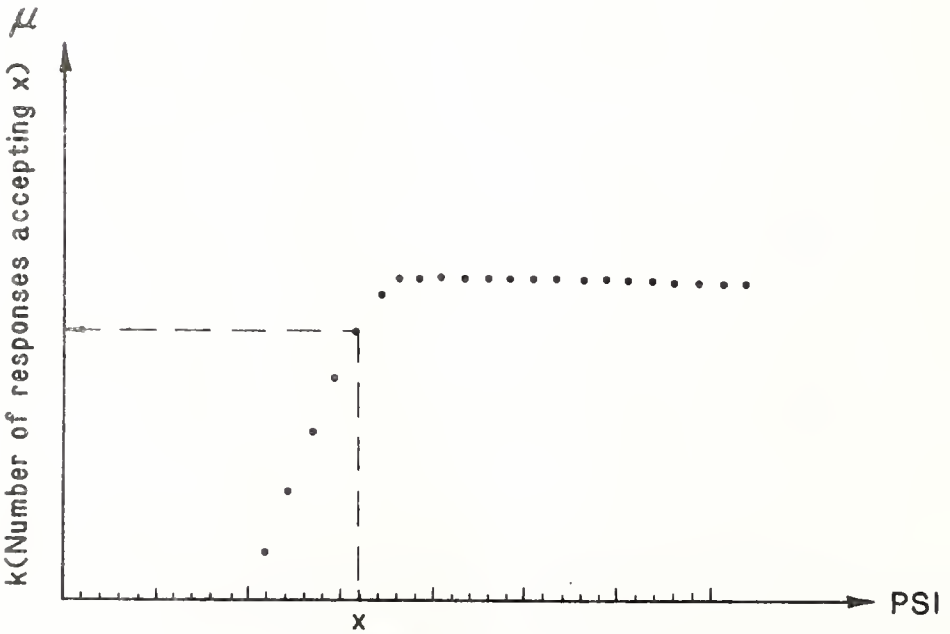


Figure 22. Acceptable serviceabiltiy range

an acceptable PSI value is proportional to the number of expert responses considering x to be in the acceptable range (Figure 22). Therefore, the membership of x is proportional to the number of responses favoring x. For each expert, the acceptable range is considered as the range of PSI values above the indicated ASI. Responses indicating a range of ASI can be handled in the following manner. If such a range covers an n number of discrete ratings, it can be considered as n different responses each indicating a single value. These "multiple responses" can then be aggregated with the others in the usual way. Since this has the effect of attaching a weight n to the considered response, a factor 1/n is attached to all its corresponding membership values to compensate for the weighting effect.

Acceptable friction number

When a pavement section is subjected to a continuous traffic load, wearing takes place, resulting in a decrease in the skid-resistance. This would continue over the years until the pavement becomes no longer safe for traffic passage. The acceptable friction number is defined as the value of friction above which the pavement does not present any skid hazard. It is common to define an unacceptable friction number as well. This is the value of friction number below which immediate rehabilitation action is needed to improve the skid properties of a pavement.

Opinions of highway engineers were sought about acceptable friction levels by means of questionnaire No. 3 (Appendix F).

These responses not only indicated the variability of opinions, but also showed some vagueness surrounding the individual opinions, since many preferred to indicate a range. Acceptable skid levels are also subjective due to the differences in judging the terminal service stage of a pavement. As in the case of serviceability levels, they can best be represented by fuzzy sets. Similarly, an acceptable friction range (Figure 23) was formed using questionnaire responses, in a manner similar to forming the fuzzy acceptable serviceability range.

In the next section it will be shown that an unacceptable friction number range and an unacceptable serviceability range also play important roles in the screening of pavement sections. With this in mind, the opinions of the highway experts were sought on the unacceptable levels as well. These responses were converted to unacceptable ranges in the same manner as above (Figure 24).

Screening of pavements

It was mentioned earlier that pavements are first scanned for roughness by using the Roadmeter. Then, the fuzzy PSI of each pavement section is compared with the fuzzy acceptable serviceability range (ASR). Extensive research has been done on decision making using fuzzy sets, and several methods have been proposed for the comparison of two fuzzy quantities. One simple method is the use of the "implication" operator (Watson et. al,1979). Let X and Y be two fuzzy propositions defined in the

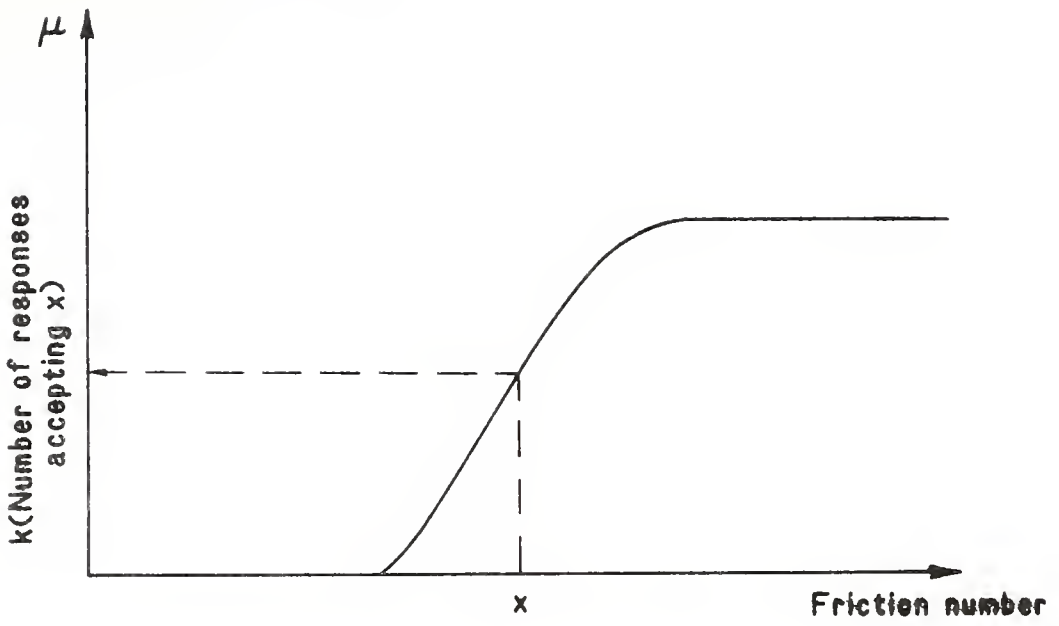


Figure 23. Acceptable friction range

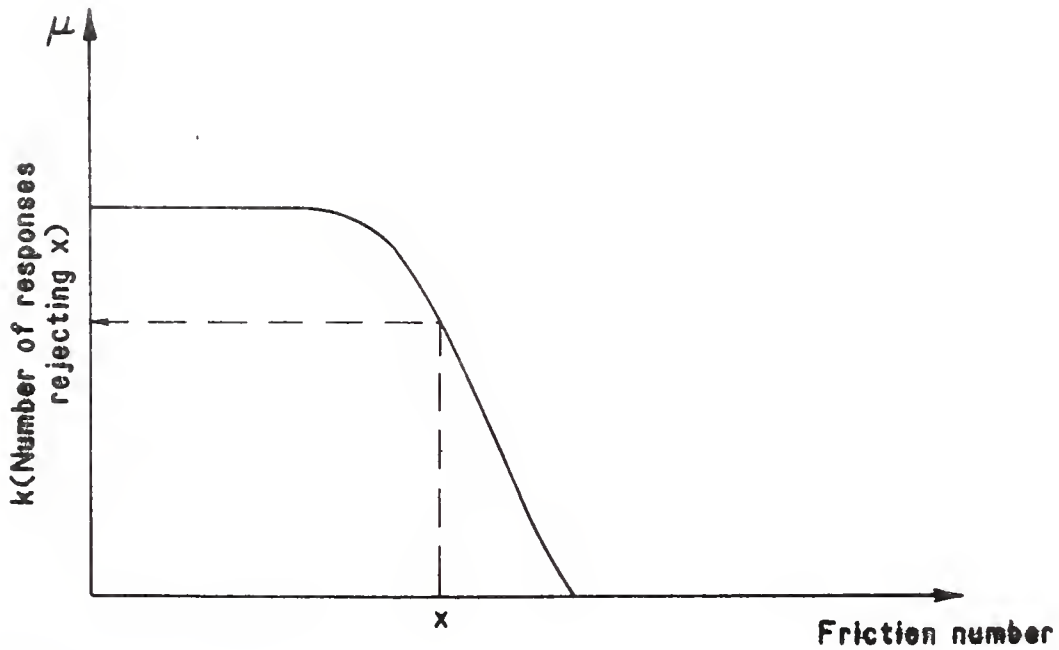


Figure 24. Unacceptable friction range

space Z by the following expressions :

$$X = \underset{z}{U}\ \mu_x(z)/z \tag{85a}$$

$$\text{and}\quad Y = \underset{z}{U}\ \mu_y(z)/z \tag{85b}$$

Then the truth value of X → Y (X implies Y) is given by :

$$\mu(X \rightarrow Y) = \min_z \max\ [1-\mu_X(z),\ \mu_Y(z)] \tag{86}$$

If X and Y are replaced by the PSI and acceptable serviceability range (ASR) respectively, then the result of equation (86), μ(PSI → acceptable range), is the truth value of the implication that PSI of a particular pavement section is acceptable. It is a number in the interval [0,1] and provides an index describing the degree of belongingness of the particular pavement section to the acceptable range. In the two preceding sections it was mentioned that an acceptable range is found more useful than an acceptable rating. This is because equation (86) can only provide a physically meaningful result if Y is replaced by ASR, and not by ASI.

To demonstrate the usefulness of the above operation the following example is considered. Let the PSI of a pavement section be given by (Figure 25) :

PSI = 0.2/2.7 + 0.75/2.8 + 1.0/2.9 + 0.67/3.0 + 0.26/3.1

and let the acceptable range be expressed by (solid line in Figure 25) :

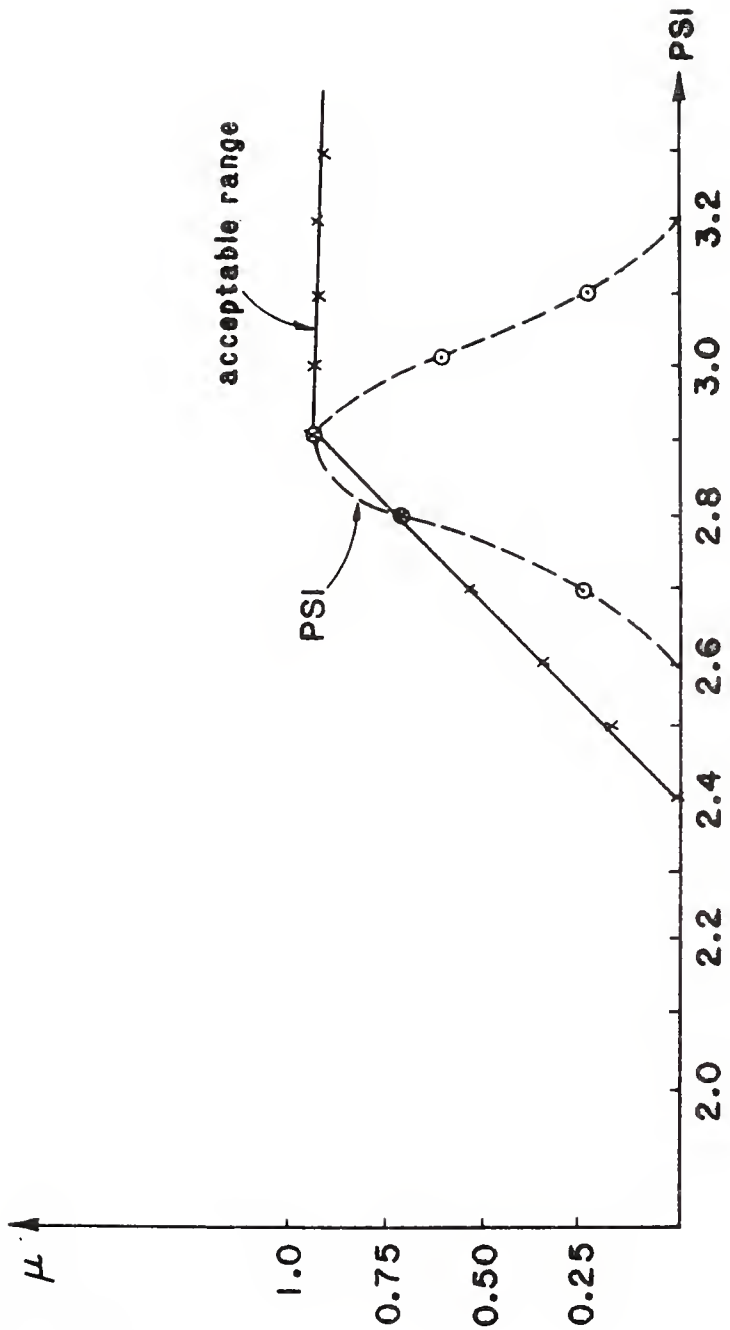


Figure 25. PSI and acceptable serviceability range

$$ASR = 0.2/2.5 + 0.4/2.6 + 0.6/2.7 + 0.8/2.8$$
$$= + 1.0/2.9 + 1.0/3.0 + 1.0/3.1 \ .....$$

The truth value of the proposition "the given PSI implies acceptability" is expressed by:

$$\mu\ (PSI \rightarrow ASR) = \min_{z} \max\ [1-\mu_{PSI}(z),\ \mu_{ASR}(z)]$$
$$= 0.8$$

This index can be conveniently used to rank order pavements according to their PSI. Sometimes there may be cases where the indices would be equal for a number of pavements. For such cases, a second method of ranking can be adopted using the unacceptable range. This results in a second index for each pavement section, which describes the degree of belonging of the section to the unacceptable serviceability range (NASR) as :

$$\mu(PSI \rightarrow NASR) = \min_{z} \max\ [1-\mu_{PSI}(z), \mu_{NASR}(z)]$$

The second index is independent of the first, since the acceptable range and the unacceptable range are not complements of each other. There is a range of PSI overlap where a direct decision of acceptability is difficult to be made.

Another notable advantage of having two indices is that they can be used to screen pavements with acceptable roughness from others. If $\mu(PSI \rightarrow \text{acceptable})$ and $\mu(PSI \rightarrow \text{unacceptable})$ are denoted by μ_A and μ_N respectively, a criterion for screening is provided by :

If $\mu_A > \mu_N$ pavement acceptable (87a)

If $\mu_A < \mu_N$ pavement unacceptable (87b)

Physically, the above criterion means that if the degree of belonging of a given PSI in the acceptable range is greater than its degree of belonging in the unacceptable range, then the corresponding pavement is acceptable from the standpoint of serviceability.

It is obvious that the same techniques are applicable when comparing friction numbers with acceptable or unacceptable friction levels.

Classification of pavement sections

After comparison with the corresponding acceptable levels, the PSI and the friction number of each section are instrumental in forming three categories of pavements as shown in Figure 26. The first category contains pavements which present traffic hazards due to inadequate skid-resistance, while pavements with unacceptable roughness constitute the second category. The third category consists of pavement sections with acceptable roughness as well as acceptable skid properties.

In routine maintenance, immediate attention is given to the first category to correct dangerous skid resistance problems. The worst performing sections which fall in to the second category are ranked for current maintenance, considering a fixed budget. Meanwhile, the pavement sections which satisfy both

FIRST CATEGORY		SECOND CATEGORY		THIRD CATEGORY	
PSI	OK	PSI	No	PSI	OK
Friction Number	No	Friction Number	N/A	Friction Number	OK
Deflection	N/A	Deflection	Measured	Deflection	N/A
Distress	N/A	Distress	Surveyed	Distress	N/A

Figure 26. Primary categorization of pavements

roughness and skid criteria at present are ranked for future rehabilitation. In order to identify the parameters relevant to ranking of the sections in each of the three categories, it is worthwhile to review the experience of several highway departments. Complete pavement management systems including effective schemes for maintenance prioritization of pavement sections, are in operation in several states. In Arizona and Arkansas, pavements are ranked based on roughness, distress rating, friction number, dynaflect measurements, and traffic count (ADT). Experience and judgment have been combined to formulate equations for a priority score. The state of Florida bases priorities on roadmeter reading, distress rating, ADT, as well as length of the pavement section and project cost. In Utah and Idaho, priorities are based on an index, FI, defined as (TRB, 1981) :

$$FI = 0.47\ [F_1\ (PSI)^{1.5} + F_2\ (SI)^{1.5} + F_3\ (DI)^{1.5}] \qquad (88)$$

where SI and DI are structural and distress indices respectively, and F_1 , F_2 ,and F_3 are weighting functions of ADT.

In the state of Washington, a number of priority rankings are assigned to all pavement sections that need rehabilitation, based on the following data.

1. Distress condition
2. Bridge condition
3. Hazardous accident locations
4. Volume-capacity ratio deficiencies
5. Geometric deficiencies

Nevada's pavement management system uses serviceability and condition data to place pavements into four broad repair classes: do nothing ; maintenance ; overlay ; and reconstruction. Priority rankings in each class are formed using PCR, ADT, maintenance cost, skid, and accident data, after weighting each of these factors according to their relative merit.

Finally, the Ohio Department of Transportation ranks pavements in three separate groups called PCRGRP, MUC, and ADT. The PCRGRP ranking, based on PCR, is considered as the first sort. MUC ranking provides a second sort based on PCR, PSI, and friction number, while ADT ranking is used only as a third sort.

The Indiana Department of Highways is currently developing an effective prioritization scheme for the second category of pavements (Figure 26). Rankings are assigned based on PSI and ADT. Further, it is also being considered to include PCR as a ranking parameter. For this new scheme to be implemented, development of an effective procedure for condition survey is vital.

Attributes for immediate maintenance

Taking into account the experience of many states, the following attributes are suggested for setting up priorities for pavements in groups 1 and 2 (Figure 26). The results of condition surveys, i.e. PCR , are of prime importance in creating priorities for the worst performing sections (group 2), since a

distress survey considers the very factors that are relevant to their rehabilitation. On the other hand, for pavement sections in group 1, skid resistance appears to be an obvious attribute for creating priorities. In addition, traffic data, represented by ADT, should play a significant role for both groups.

It was stated that priorities for immediate maintenance are probably created under a fixed budget. Hence, when assigning priorities, the decision makers may prefer to complete the maximum number of "jobs" when working under budget constraints. An approximate estimate of cost would be adequate to include cost in the prioritization scheme. It is possible to obtain such an estimate from maintenance experts, given the relevant performance data. For this purpose, skid data for the first category, and condition and dynaflect data for the second category, would be sufficient. It should be noted that PSI is involved in the scheme only as a means for determining to which category the section belongs. The foregoing discussion on the priority model is mathematically summarized in what follows.

First category

For pavements which do not meet acceptable skid criteria :

$$\text{priorities} = f_1 (FN, ADT, \text{appr.relative cost})$$

$$\text{relative cost} = g_1 (\text{friction number})$$

$$\Rightarrow \text{priorities} = F_1 (f, a) \qquad (89)$$

Second category

For pavements which do not meet acceptable roughness criteria :

$$\text{priorities} = f_2 (\text{PCR, ADT, appr. relative cost})$$

$$\text{relative cost} = g_2 (\text{PCR , deflection data})$$

$$\Rightarrow \text{priorities} = F_2 (p_c, a, \delta) \qquad (90)$$

Equations (89) and (90) show that the assignment of priorities by decision makers, results basically from subjective consideration of the relative magnitudes of relevant performance and traffic data.

Attributes for future maintenance

Pavement sections in group 3 will deteriorate with time, with respect to rideability and skid-resistance, and eventually reach a terminal (unacceptable) serviceability or friction level. Therefore, it is appropriate to consider these sections for future rehabilitation. The times to reach these limits are known as serviceability life and friction life, respectively. These two service lives can be used for creating priorities for future rehabilitation.

In what follows, two deterministic procedures that are presently used to predict service lives in Indiana are first reviewed. Later in the discussion it will be shown how these two basic procedures can be extended to obtain service lives of pavement sections characterized by fuzzy performance values.

Determination of PSI life

The most common method used by transportation engineers to determine the PSI life of a pavement is the AASHO (1972) method.

Deterioration of the serviceability of a pavement section with time depends on many factors. Traffic load applied on the pavement, structural geometry of the pavement, and present serviceability are the dominant factors. Of these, traffic load is the most complicated, as normal highway traffic is a random combination of vehicles with different axle loads and numbers of axles. The procedure used in the AASHO Design Guide (1972) is to convert the varying axle loads to a common denominator and express traffic as the sum of the converted axle loads. The common denominator is an 18 kip single axle load. Traffic load is thus expressed as an equivalent 18 kip single axle load (ESAL).

The structural geometry of the pavement is expressed in terms of a structural number (SN). The following equation relates the thickness of each layer to the structural number :

$$SN = a_1 D_1 + a_2 D_2 + a_3 D_3 \qquad (91)$$

where D_i is the thickness of the i^{th} layer (surface, base, or subbase), and a_i is a layer coefficient, which is a measure of the relative ability of the i^{th} layer material to function as a structural component of the pavement. Typical values of the coefficients a_i used in AASHO Road Test pavements are found in Table 7.

Table 7. Typical values of layer coefficients

Asphalt concrete surface course	0.44
Crushed stone base course	0.14
Sandy gravel base course	0.11

In developing its empirical procedure, the AASHO Interim Guide (1972) assumes the initial PSI of any pavement section to be 4.2. Then the general AASHO Road Test equation for flexible pavements is :

$$\log \frac{4.2 - p_t}{4.2 - 1.5} = \beta \left(\log W_{t_{18}} - \log \rho \right) \tag{92}$$

$$\text{For 18 kip. ESAL,} \quad \beta = 0.40 + \frac{1094}{(SN + 1)^{5.19}} \tag{93}$$

$$\text{and,} \quad \log \rho = 9.36 \log (SN + 1) - 0.20 \tag{94}$$

where $W_{t_{18}}$ is the accumulated ESAL up to time t, p_t is the PSI value at time t, and SN is the structural number of the pavement. Similar equations are available for rigid pavements as well (AASHO Interim Guide, 1972). A method to calculate the projected ESAL ($W_{t_{18}}$) is discussed in Yoder and Witczak (1975). Knowing the percentage of trucks on the road and the factor that converts truck-load to 18 kip single axle load, i.e the ESAL per truck, they recommmend the following equations to obtain $W_{t_{18}}$:

$$ESAL_0 = \text{ADT x Percent trucks x ESAL per truck} \tag{95}$$

$$W_{t_{18}} = \int_0^t ESAL_0 \left(1 + i \right)^T dT \tag{96}$$

where $ESAL_0$ is the initial equivalent axle load applications per day and i is the annual traffic growth. The percentage of trucks is obtained from either traffic counts or available past

data. $W_{t_{18}}$ represents traffic in all lanes (in a multi-lane facility) and in both directions. Most states use a directional distribution factor of 0.5 and a lane distribution factor of 0.8-1.0. Knowing the structural number of a pavement and the ESAL, equations (91)-(94) can be used to determine the PSI value at time t.

As an example, data obtained for a section of a state road in Indiana was used to predict the change of PSI with time. The structural geometry of this section is indicated in Figure 27. If the traffic count is 1200 vehicles per day (both directions), using equation (91) and a_i values from Table 7, the value of SN is equal to 3.0 and from equations (93) and (94), β and log ρ are obtained as 1.22 and 5.44, respectively.

The following assumptions are usually made regarding the percentage of trucks and the ESAL per truck:

Percent. of trucks : 20-30% on interstates and 8-12% on others

ESAL per truck : 0.6-0.7 on interstates and 0.45 on others

By using 10%, 0.45, and 4% for percent trucks, ESAL per truck and the annual growth rate, respectively, the initial axle load is :

$$ESAL_0 = 54.0$$

Equation (96) can now be solved to obtain the values of $W_{t_{18}}$ (Table 8). Finally, equation (92) is used to compute the

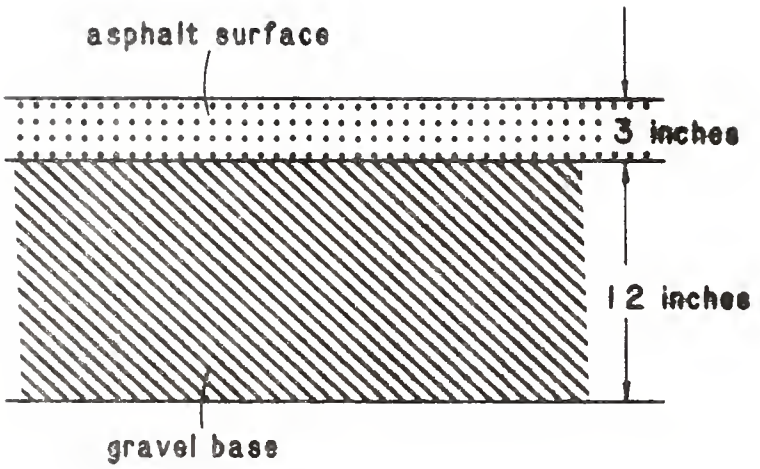


Figure 27. Structural components of a sample pavement section

Table 8. Variation of PSI with time

t (yrs.)	$W_{t_{18}}$	PSI
1	20104.2	4.089
2	20892.6	3.940
3	21681.0	3.760
4	22469.4	3.560
5	23257.8	3.330
6	24046.2	3.100

PSI values at different times, which are plotted in Figure 28. This curve could be used to predict the PSI life of the pavement sections. Actually, additional uncertainties are involved in the procedure based on equations (92) to (94) such as that surrounding traffic counts. Moreover, uncertainty is also introduced through the assumptions. In spite of these, a one to one performance - time relationship is assumed in this work for simplicity.

Determination of friction life

The problem of deterioration of skid-resistance of a pavement section with time is less complicated than that of reduction in serviceability. Cumulative traffic (ADT) is the only factor that influences the former phenomenon.

Research conducted by the Research and Training Center of the Indiana Department of Highways has produced the following general equation (IDOH report, 1982), which expresses variation of skid-properties of any kind of pavement with traffic and seasonal variations :

$$FN = \beta_o + \beta_1 XL + \beta_2 SPI + \beta_3 SUI \qquad (97)$$

where,
$$XL = \log_{10} \left(\sum_{0}^{t} ADT \right) \qquad (98)$$

SPI = spring factor

SUI = summer factor

SPI will take the value of 1 if measurements are made in spring

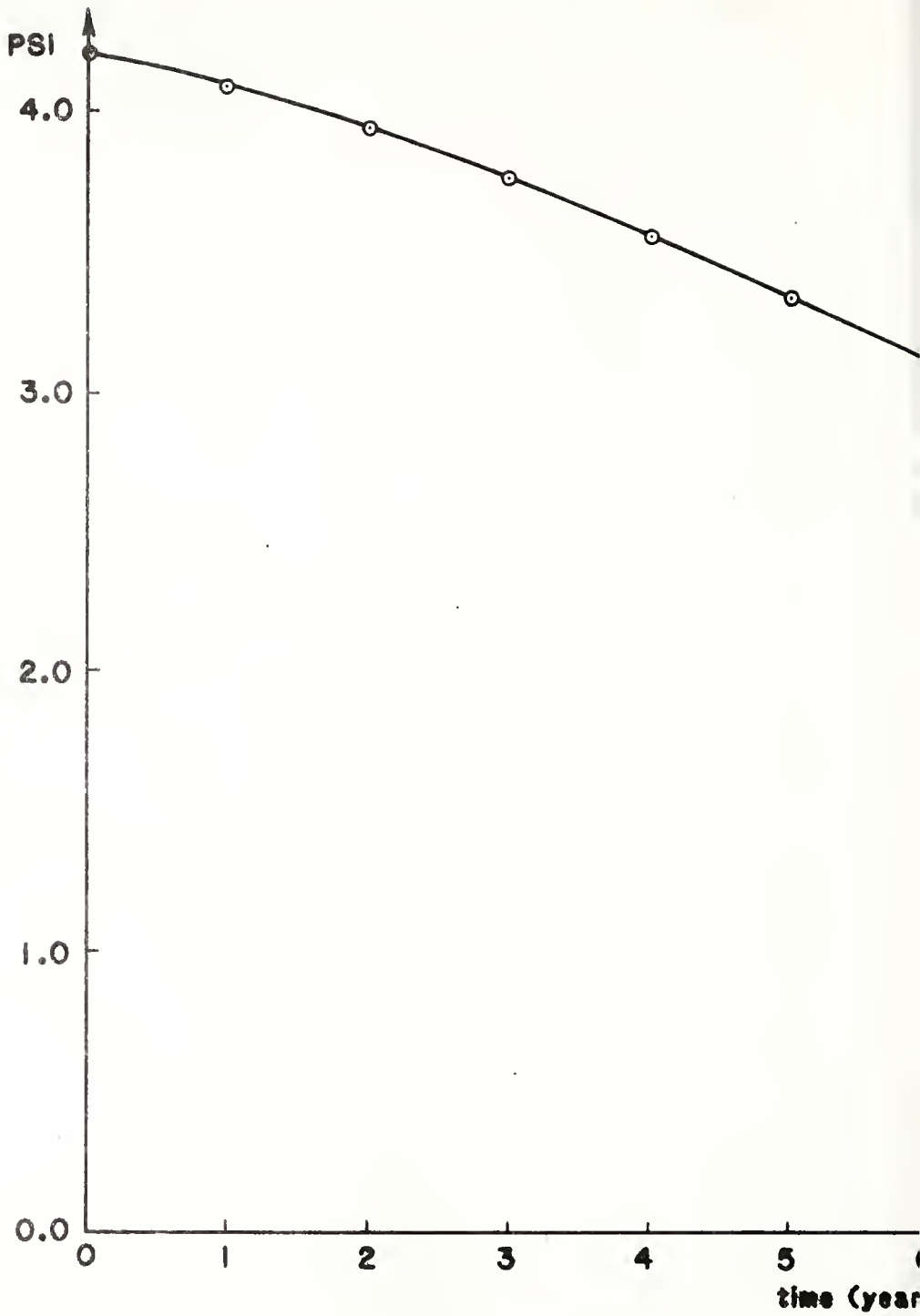


Figure 28. PSI vs time curve for the sample section

and 0 otherwise. Similarly, SUI will take the value of 1 if measurements are made in summer and 0 otherwise.

Friction life of a pavement is determined by the non-recoverable reduction in skid-resistance, and hence, seasonal variations of the friction number will not be considered here. When the non-recoverable reduction in skid-resistance is considered,

$$\beta_2 = \beta_3 = 0.0$$

Therefore, equation (97) is modified to :

$$FN = \beta_o + \beta_1 \, XL \tag{99}$$

Typical values of β_o and β_1 for any kind of pavement were obtained from statistical correlations (IDOH report ,1982).

As an example, the FN vs. time relationship was obtained for the asphaltic section shown in Figure 27, with β_0 and β_1 equal to 152.5 and -16.512, respectively (IDOH report, 1982). Assuming a 4% growth rate, equation (99) yields the FN vs. time curve shown in Figure 29. This type of curves enable the prediction of future skid properties of any pavement section.

It is worthwhile to comment on equation (97) in the light of the applications of fuzzy sets techniques to the pavement management system. Seasonal variations of the friction number are incorporated as imprecision introduced in the friction measurements due to changes in temperature and rainfall (see Chapter 3). Thus, the factors β_2 and β_3 were assumed to be zero in deriving

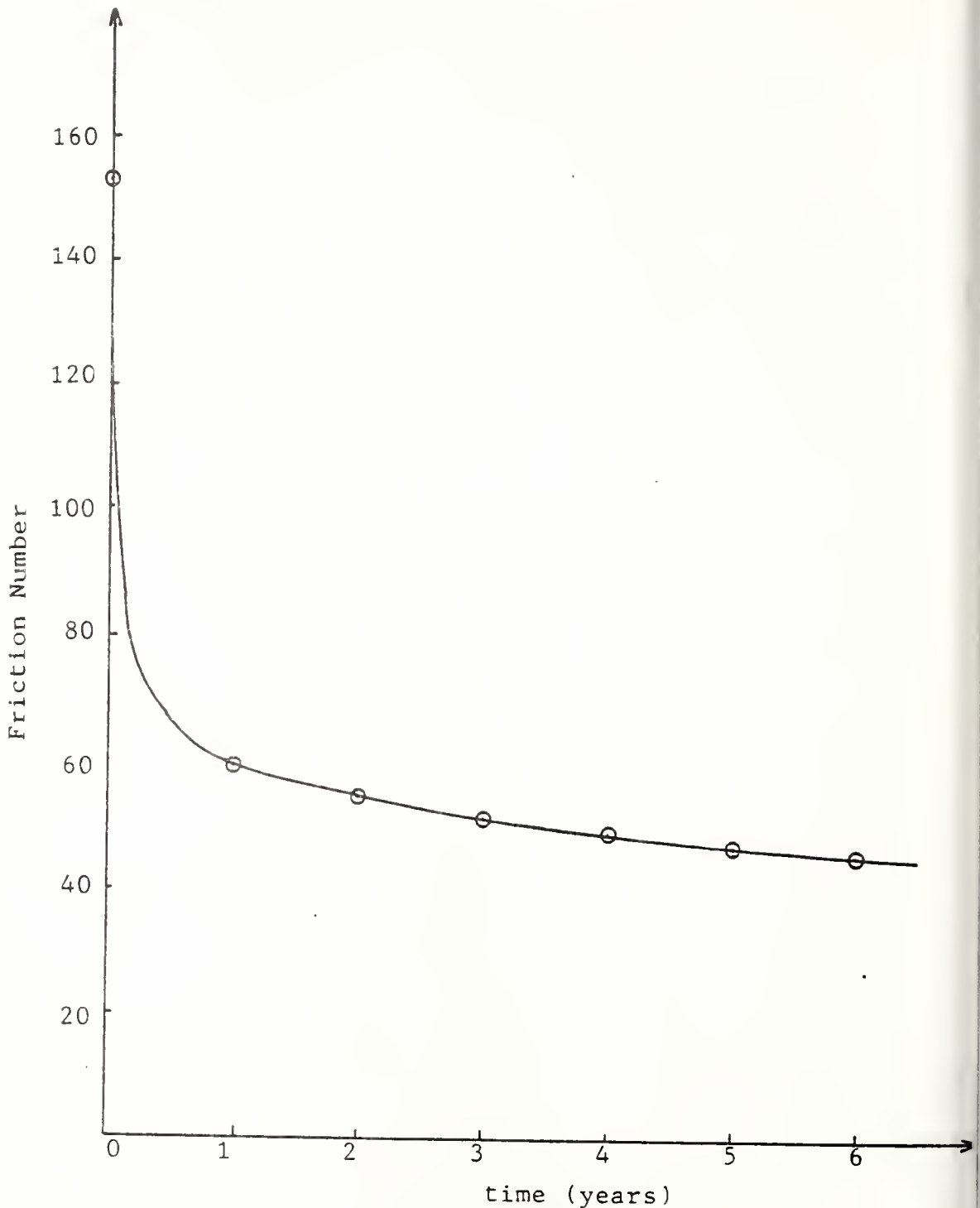


Figure 29. Friction number vs time curve for the sample section.

equation (99) to make it compatible with the proposed method. At the present stage, a method which predicts the irrecoverable decrease in skid-resistance with time would suffice. It is in this respect that equation (99), without the seasonal variation factors, basically meets the present need. Later, it will be shown how fuzzy sets methods could be extended to the prediction of service lives of pavements, using the basic equations for serviceability (equations (92)-(94)) and skid-resistance (equation (99)).

Prediction of service lives

Terminal performance levels such as acceptable serviceability index (ASI) and acceptable friction number are vaguely defined. Therefore, these levels are represented by fuzzy numbers. Some degree of vagueness also appears in the pavement serviceability index (PSI) due to panel members' subjective opinions as well as the uncertainty in RR. In view of this, the concept of fuzzy PSI has been inroduced in place of the conventional PSI. Further, the Skid-tester measurements are influenced by climatic conditions as well as changes in vehicle speed. It was shown in Chapter 4 that this problem can be overcome by representing the friction number by a fuzzy number with a suitable tolerance interval.

According to the previous definition, the time to reach the terminal performance level is the service life of a pavement with respect to a particular pavement property. Since the performance

levels and the acceptable and unacceptable levels are vague and are represented by fuzzy sets, it follows that the service lives will also be fuzzy quantities. The method of extracting the fuzzy service lives for serviceability or skid-resistance, from performance vs. time curves (Figures 28 and 29), will now be illustrated.

Let us assume that the present value of any pavement property is represented by the following fuzzy set :

$$P = \underset{i}{U}\ \mu_P (p_i) / p_i \qquad (100)$$

where p_i is any value of the pavement property.

Since the relation between P and time, t, for a particular section is assumed to be a one to one relationship (see the section on prediction of PSI life), the coresponding time can be found as :

$$T_p = \underset{i}{U}\ \mu_p (p_i) / t_i \qquad (101)$$

where t_i is the time corresponding to p_i (Figure 30).

Similarly, if the unacceptable performance level is denoted by :

$$U = \underset{j}{U}\ \mu_u (p_j) / p_j \qquad (102)$$

the time at which the pavement becomes unacceptable can be obtained as :

$$T_u = \underset{j}{U}\ \mu_u (p_j) / t_j \qquad (103)$$

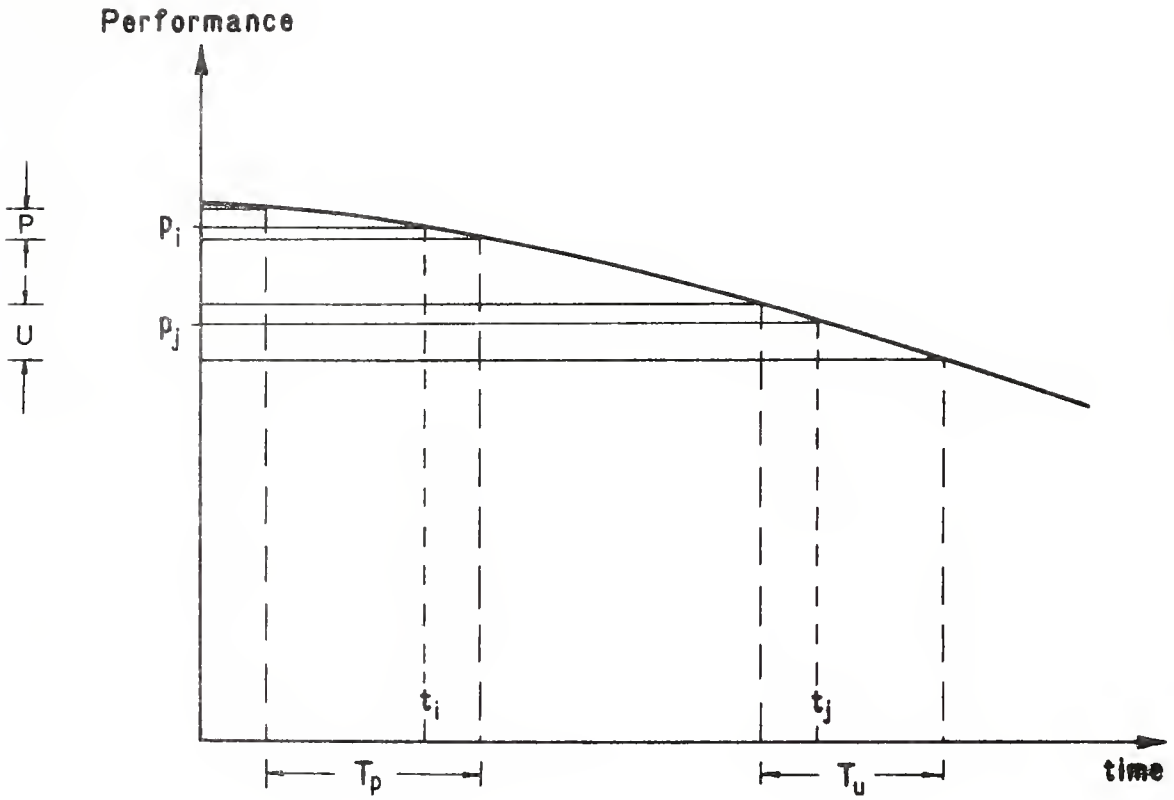


Figure 30. A typical performance vs time curve

where t_j is the time corresponding to p_j (Figure 30).

By definition, the service life (T) is expressed by the subtraction operation :

$$T = T_u - T_p \tag{104}$$

and $T = \underset{k}{U}\ \mu_T (t_k) / t_k$

with (from extension principle) :

$$\mu_T (t_k) = \underset{t_k = t_j - t_i}{\text{Sup}} \min [\mu_P (p_i) , \mu_U (p_j)] \tag{105}$$

For example, let the current PSI of a certain flexible pavement section belonging to group 3 be given by the following fuzzy set :

$P = 0.8/4.1 + 1.0/4.0 + 0.8/3.9 + 0.6/3.8$

and the unacceptable limit be :

$U = 1.0/3.2 + 0.8/3.3 + 0.6/3.4$

Then, Figure 28 gives :

$T_p = 0.8/1.0 + 1.0/1.6 + 0.8/2.2 + 0.6/2.6$

and

$T_u = 1.0/5.5 + 0.8/5.2 + 0.6/4.9$

From equation (105) :

$$T = 0.8/4.5 + 0.8/4.2 + 1.0/3.9 + 0.8/3.6 + 0.8/3.3 + 0.8/3.0 + 0.6/2.9 + 0.6/2.7 + 0.6/2.6 + 0.6/2.3$$

This is the fuzzy PSI life for the particular section which can be used as one attribute value in establishing the priority for this section. By programming equation (105), the service lives of any pavement section (with respect to PSI and friction), can be computed as fuzzy sets. The computer program LIFE is used for this computation. Performance (P) vs. time (t) relationship is used as an input to this program.

Friction and PSI lives are two relevant attributes for creating priorities for pavements in group 3. The ADT count appears to be another attribute, but due to the inclusion of ADT in the development of PSI vs. time and friction number vs. time curves, it will not be an independent attribute. Any influence due to the changes in ADT are taken into account indirectly by service lives which are functions of ADT. Thus, it is appropriate to consider only two attributes for group 3, the two types of service lives. This can be conveniently expressed in a mathematical form similar to equations (88) and (89) :

$$\text{priorities} = F_3(T_{PSI} , T_{FN}) \qquad (106)$$

Equations (88), (89) and (106) show the relevant attributes for groups 1, 2 and 3, respectively. These equations also show that, in formulating a decision model for creating priorities, a common scheme can be adopted for all three pavement categories despite the different attributes involved. Therefore, the modelling discussed in the following, entails a general set of attributes.

Fuzzy decision making techniques

The state of any pavement can be represented by n attributes A_i (i=1,n) with respect to maintenance needs, which represent performance or traffic data. Thus, a generalized form of equations (88) ,(89) and (106) will be :

$$\text{priorities} = F (A_i) \tag{107}$$

As discussed in the previous chapters, pavement performance data inherit human and system uncertainties, making the fuzzy representation a convenient and realistic approach. Then, the performance attributes in equation (107) can be defined by the following fuzzy set :

$$A_i = \underset{j}{U} \mu_i (a_j) / a_j \tag{108}$$

where a_j represents any value on the scale of A_i. Similar expressions can be written for all m pavement sections of the particular category. Therefore, the priority of any k^{th} section (k=1,m) can be written in the following form :

$$R_{pk} = F (A_{ik}) \tag{109}$$

with

$$A_{ik} = \underset{j}{U} \mu_{ik} (a_j) / a_j \tag{110}$$

where A_{ik} denotes the performance of the k^{th} section with respect to the i^{th} attribute.

The maintenance urgency of this section with respect to the others can be visualized as a priority utility u_k on a standard scale. Then, the decision making problem reduces to one of maximization of u_k for k = 1, m.

It is possible to express a set of n attribute values by the n-tuple $\langle a_{1p},.....,a_{nq}\rangle$, where a_{1p} and a_{nq} are the p^{th} and q^{th} values of the first and the n^{th} attributes, respectively. An expert knowledge base which uniquely defines a utility value for each n-tuple can be constructed from the responses of the experts to a number of questions such as : "If the PCR is 70.0 and the dynaflect reading is 0.001 inches for an unacceptably rough pavement with an ADT of 3000, what relative priority value would you assign on a scale of 1-10 ?". An expert can assign such a subjective value based on heuristic rules that have come through years of pavement management experience. Thus, for instance, her or his opinion could be that for an unacceptably rough pavement with a high PCR, relatively low deflection, and low traffic, the maintenance priority is low and she or he might assign a utility value of 3.0 in a utility scale of 1 to 10. Questionnaires have been prepared (Nos. 6 and 7 in the Appendix F) to obtain these utilities for all three pavement groups.

Such a procedure would be more realistic if it would involve a number of experts' subjective opinions. Consequent to this, the expert knowledge base will contain a set of fuzzy utility values associated with each n-tuple. Mathematical techniques involved in building the expert knowledge base for priority utilities will

be discussed in more detail later in the chapter.

The next step involves formation of memberships of the n-tuples. If a_{ip} is any general element in the fuzzy attribute set A_{ik}, then $\langle a_{1p},.....,a_{nq}\rangle$ is any general element in the fuzzy set A_{1k} X A_{2k} X ..... X A_{nk} where "X" indicates the cartesian product. This follows from the development of cartesian product operation in Chapter 2 (equations (13) to (17)) :

$$\text{If} \qquad A_k = A_{1k} \text{ X } A_{2k} \text{ X } \ldots\ldots\ldots A_{nk} \tag{111}$$

$$\text{then,} \qquad A_k = \underset{j}{U}\ \mu_{jk}/\langle a_{1p},.....,a_{nq}\rangle \tag{112}$$

$$\text{where} \qquad \mu_{jk} = \min\left[\mu_{A_{1k}}(a_{1p}),....,\mu_{A_{nk}}(a_{nq})\right] \tag{113}$$

It is recognized that μ_{jk}, the membership value of the j^{th} n-tuple $\langle a_{1p},.....,a_{nq}\rangle$ for section k, represents the link between the k^{th} section and the j^{th} n-tuple.

The fuzzy utility, u_j , associated with each n-tuple $\langle a_{1p},.....a_{nq}\rangle$ can be obtained from the expert knowledge base mentioned above. If this is denoted by :

$$U_j = \underset{1}{U}\ \mu_{1j}/u_1 \tag{114}$$

the link between the j^{th} n-tuple and the priority value u_1 is given by μ_{1j}. Hence the link between the k^{th} section and u_1 is determined by μ_{1j} and μ_{jk}. This is analogous to the overall strength of a chain, which is the lowest of the strengths of the individual links. Therefore, equations (112) and (114) can be

combined using the "min" operator to find the relative priority of the k^{th} section as :

$$R_{pk} = \bigcup_j \mu_{jk} \wedge \bigcup_l \mu_{lj}/u_l \qquad (115)$$

Thus, the relative priority of each section is a fuzzy set taking values in the interval 1-10. This scale is used for convenience, as it is presently used in Indiana for assigning non-fuzzy priorities.

Once relative priorities of all sections have been obtained from equation (115), they must be ranked. A method of ranking fuzzy utilities will be introduced in the course of illustrating a numerical example. The purpose of the example is to let the reader go through the steps involved in the proposed ranking method. Let us consider three pavement sections belonging to the second category with properties shown in Table 9.

By inspection, it is obvious that the priority order for maintenance of these sections is simply 1, 2, and 3. The example is intentionally kept simple so that the same priority order can be obtained using the fuzzy model, with minimum computations. The procedure can be applied to actual, more complex cases by using the computer programs developed to perform the different operations.

In the case of section 1, the cartesian product of the different attributes (equation (113)), results in the following fuzzy set.

Table 9. Attribute values for the section

Property	Section 1	Section 2	Section 3
ADT	10,000	6,000	3,000
PCR	0.8/19.0 + 1.0/20.0 + 0.8/21.0	0.8/39.0 + 1.0/40.0 + 0.8/41.0	0.8/79.0 + 1.0/80.0 + 0.8/81.0
Deflection (10^{-13} in)	0.6/9.9 + 1.0/10.0 + 0.6/10.1	0.6/5.9 + 1.0/6.0 + 0.6/6.1	0.6/1.9 + 1.0/2.0 + 0.6/2.1

0.6/<10000,19.,9.9> + 0.8/<10000,19.,10.0> + 0.6/<10000,19.,10.1>

+ 0.6/<10000,20.,9.9> + 1.0/<10000,20.,10.0> + 0.6/<10000,20.,10.1>

+ 0.6/<10000,21.,9.9> + 0.8/<10000,21.,10.0> + 0.6/<10000,21.,10.1>

The expert knowledge base will contain crisp or fuzzy utility values for each of the above 3-tuples. Shown in Table 10 are an extracts from such an expert system. For instance, as a special case of equation (114), the utility corresponding to the 3-tuple < 10000, 19.0, 9.9 > is :

$$U = 1.0/9.6 + 0.8/9.5$$

Next, all such utilities can be combined with the memberships of the corresponding 3-tuples using equations (115), to obtain the fuzzy priority for the first section as :

$$R_{pl} = 0.6\hat{}(1.0/9.6 + 0.8/9.5) + 0.8\hat{}(1.0/9.7 + 0.8/9.6)$$

$$+ 0.6\hat{}(1.0/9.8 + 0.8/9.7) + 1.0\hat{}(0.9/9.6 + 0.7/9.5)$$

$$+ 0.6\hat{}(0.9/9.5 + 0.7/9.4) + 0.6\hat{}(0.9/9.7 + 0.7/9.6)$$

$$+ 0.6\hat{}(0.8/9.4 + 0.6/9.3) + 0.8\hat{}(0.8/9.5 + 0.6/9.4)$$

$$+ 0.6\hat{}(0.8/9.6 + 0.6/9.5)$$

$$R_{pl} = 0.6/9.3 + 0.6/9.4 + 0.8/9.5 + 0.9/9.6 + 0.8/9.7 + 0.6/9.8$$

The relative priority rankings for sections 2 and 3 also can be

Table 10. A sample expert knowledge base

ADT = 3000

PCR Def. (mil. in)	79.0	80.0	81.0
1.9	1.0/3.6 + 0.8/3.5	0.9/3.5 0.7/3.4	0.8/3.4 0.6/3.3
2.0	1.0/3.7 + 0.8/3.6	0.9/3.6 + 0.7/3.5	0.8/3.5 + 0.6/3.4
2.1	1.0/3.8 + 0.8/3.7	0.9/3.7 + 0.7/3.6	0.8/3.6 + 0.6/3.5

ADT = 6000

PCR Def. (mil.in.)	39.0	40.0	41.0
5.9	1.0/6.6 + 0.8/6.5	0.9/6.5 + 0.7/6.4	0.8/6.4 + 0.6/6.3
6.0	1.0/6.7 + 0.8/6.6	0.9/6.6 + 0.7/6.5	0.8/6.5 + 0.6/6.4
6.1	1.0/6.8 + 0.8/6.7	0.9/6.7 + 0.7/6.6	0.8/6.6 + 0.6/6.5

ADT = 10000

PCR Def. (mil. in)	19.0	20.0	21.0
9.9	1.0/9.6 + 0.8/9.5	0.9/9.5 + 0.7/9.4	0.8/9.4 + 0.6/9.3
10.0	1.0/9.7 + 0.8/9.6	0.9/9.6 + 0.7/9.5	0.8/9.5 + 0.6/9.4
10.1	1.0/9.8 + 0.8/9.7	0.9/9.7 + 0.7/9.6	0.8/9.6 + 0.6/9.5

obtained as below in a similar fashion :

$R_{p2} = 0.6/6.3 + 0.6/6.4 + 0.8/6.5 + 0.9/6.6 + 0.8/6.7 + 0.6/6.8$

$R_{p3} = 0.6/3.3 + 0.6/3.4 + 0.8/3.5 + 0.9/3.6 + 0.8/3.7 + 0.6/3.8$

Ordering of priorities

The fuzzy priority for any section k (equation 115) can be expressed in a more general form (Figure 31) as :

$$R_{pk} = \bigcup_j \mu_k(u_j)/u_j \qquad (116)$$

where u_j represents any value on the utility scale.

The fuzzy form of R_{pk} cannot be used directly to order priorities for pavement sections. From the above equation it is impossible to select a single u_j as the priority, since different degrees of uncertainty ($\mu_k(u_j)$) are associated with each u_j. Jain (1976) suggested a procedure to overcome this difficulty.

In extending Jain's work (1976) to the present case, first, a maximizing set for the section k is defined as (Figure 31) :

$$R_{mk} = \bigcup_j \mu_k'(u_j)/u_j \qquad (117)$$

where $$\mu_k'(u_j) = u_j/u_{max} \qquad (118)$$

and $$u_{max} = \sup_{j,k} (u_j) \qquad (119)$$

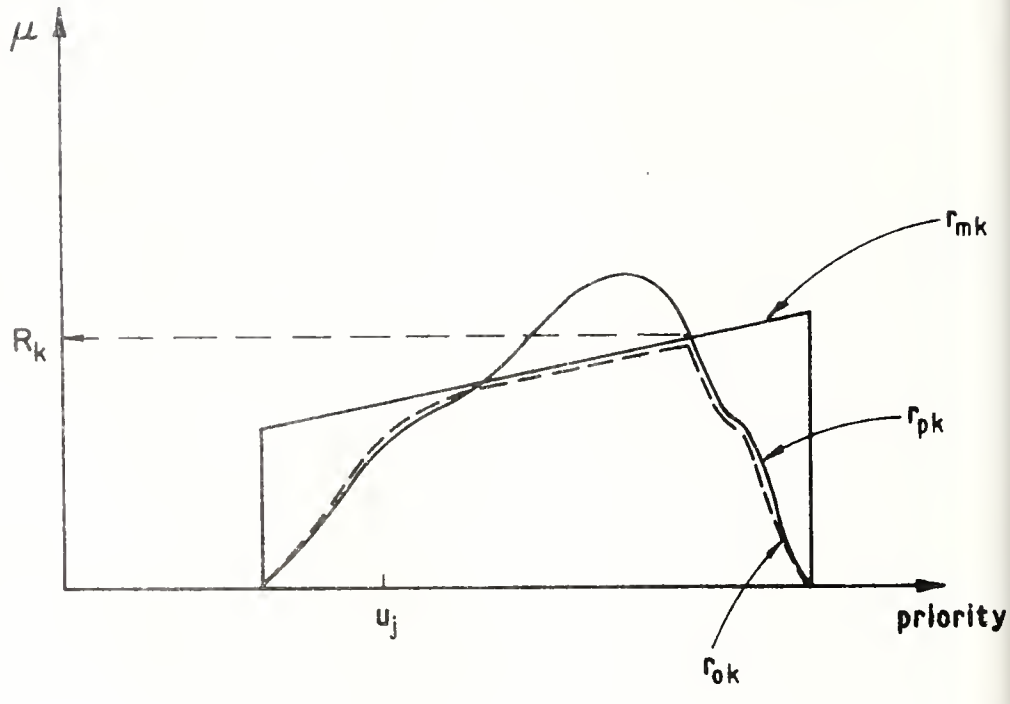


Figure 31. Ordering of fuzzy priorities

The value u_{max} is the maximum priority in the support domain of all sections. The membership of u_j in R_{pk} (equation 116) originates from the uncertain nature of performance data and the subjectivity involved in decision making. On the other hand, the membership of u_j in R_{mk} (equation 117) represents the relative magnitude of each u_j with respect to the highest priority value in the support domain of all sections. Therefore, the combination of R_{pk} and R_{mk} yields a fuzzy set whose membership depends on both priority and uncertainty. The resulting fuzzy set is known as the optimizing fuzzy set for section k, and since it assumes credibility in both sets of information, the intersection operation is used for the aggregation of R_{pk} and R_{mk} (Figure 31) :

$$R_{ok} = R_{mk} \cap R_{pk} \tag{120}$$

$$= \bigcup_j \mu_k'(u_j) \wedge \mu_k(u_j)/u_j \tag{121}$$

$$= \bigcup_j \mu_k''(u_j)/u_j \tag{122}$$

Following the ranking technique proposed by Jain (1976), the relative rank of the k^{th} section is defined by :

$$r_k = \sup_j \mu_k''(u_j) \tag{123}$$

By applying equations (117) to (119) to the previous example, the following maximizing sets are obtained :

$$R_{m1} = 0.93/9.3 + 0.94/9.4 + 0.95/9.5 + 0.96/9.6 + 0.97/9.7 + 0.98/9.8$$

$$R_{m2} = 0.63/6.3 + 0.64/6.4 + 0.65/6.5 + 0.66/6.6 + 0.67/6.7 + 0.68/6.8$$

$$R_{m3} = 0.33/3.3 + 0.34/3.4 + 0.35/3.5 + 0.36/3.6 + 0.37/3.7 + 0.38/3.8$$

And, according to equations (120) to (122) :

$$R_{o1} = 0.6/9.3 + 0.6/9.4 + 0.8/9.5 + 0.9/9.6 + 0.8/9.7 + 0.6/9.8$$

$$R_{o2} = 0.6/6.3 + 0.6/6.4 + 0.65/6.5 + 0.66/6.6 + 0.67/6.7 + 0.6/6.8$$

$$R_{o3} = 0.33/3.3 + 0.34/3.4 + 0.35/3.5 + 0.36/3.6 + 0.37/3.7 + 0.38/3.8$$

Finally, the relative ranks are given by the respective highest membership values : 0.9, 0.67, and 0.38. Consequently, as expected, the priority order for maintenance will be 1,2, and 3.

One can foresee a situation where the highest memberships of the optimizing sets would be the same for many sections. Such a situation can be handled by modifying equations (117) to (119), in defining a minimizing set. If 1 denotes such a section where the maximizing criterion fails :

$$R_{(min)1} = \bigcup_j \mu_1'(u_j)/u_j \qquad (124)$$

where $\mu_1'(u_j) = u_{min}/u_j$ (125)

and $u_{min} = \inf_{j,1} (u_j)$ (126)

Then, on repeating the procedure indicated by equations (120) to (122), a second set of rankings can be obtained for these sections. This provides a second criterion to rank the sections for which the first criterion is insufficient to make a differentiation.

An alternative and more interactive method for the combination of R_{pk} and R_{mk} exists where the algebraic product replaces the intersection operation (see Connectives, Chapter 2). When this is performed on the numerical example considered here, different relative rankings are obtained as : 0.86, 0.59, and 0.32. The priorities retain the 1,2, and 3 order. However, the min-max operations which are consistent with fuzzy logic are used in this work.

Expert knowledge base for maintenance priority

The key to the decision making techniques introduced previously is the expert knowledge base obtained in terms of maintenance priorities. This base should contain a crisp or a fuzzy utility (relative priority) value for each possible combination (tuple) of attribute values. Mathematically, if a_{ij} is any value of the i^{th} attribute A_i, each n-tuple $< a_{1p}, \ldots\ldots a_{nq} >$ will define a priority utility u in the

expert knowledge base for decision making.

Different experts may have different priorities for rehabilitation of deficient pavement sections. Moreover, even an individual expert may have an uncertainty about the utility that he allocates. Hence, in general, the utility will be a fuzzy number. Two ways of obtaining utility values using expert knowledge, are now discussed.

Interval (direct) method

As there can be a large number of n-tuples of the form $< a_{1p},....,a_{nq} >$ covering the domains of all attribute values, it is practically impossible to obtain utility values for each of them from the experts. The interval method is an approximate, but simple, method of achieving this in practice. As a first step, it involves separating the domain of each attribute into a number of working intervals. These intervals are determined in such a way that the maintenance urgency does not vary substantially within them. Experience of various highway departments will be helpful here. For example, the Indiana Department of Highways is presently using ADT ranges of 0-1000, 1000-3000, 3000-6000, 6000-10000, and greater than 10000, in its ranking procedure. Similar intervals can be suggested for PCR, deflection, and friction number.

The higher the impact of a certain attribute on maintenance urgency, the smaller should be the size of such an interval for that attribute. Further, if the intervals are made sufficiently

small, the number of pavement sections falling in the same interval for all three attributes can be minimized. These two restrictions can also be helpful in defining appropriate intervals.

For example, if pavements of group 2 are considered, the PCR of any section will be low, since these pavements are the worst performing. Therefore, a PCR working range of 0-75 can be established for this category. If five intervals of PCR (0-15, 16-30, 31-45, 46-60, 61-75), and two intervals for the deflection (0-1.0 mils, >1.0 mils) are selected, 50 (5X5X2) values of utilities are sufficient to apply the interval method. Although the method is analytically simple, providing 50 or more utilities can be cumbersome for highway personnel. This is an obvious drawback of this method.

Each utility assigned by experts will have some fuzziness surrounding it. This can be overcome by instructing them to provide a utility range for a particular tuple interval; this will be more realistic than a crisp number. A method to convert such intervals assigned by each expert into fuzzy sets has been discussed in the section on the acceptable serviceability.

Interpolation (indirect) method

Although this method stems from the interval method, it is a more accurate method of building the expert system. Judgments of experts are sought at a minimum number of attribute value combinations, covering a wide range. Then, a point by point polynomial interpolation is performed to obtain fuzzy utilities at any

combination (n-tuple) of attribute values. Although analytically more complex, this certainly provides less burden for the experts, while producing a unique fuzzy utility for each different n-tuple of attribute values.

The underlying concept is that a utility assigned by an expert should vary continuously with respect to each attribute. This is likely since experts are not expected to assign utilities in a haphazard manner. A specific interpolation technique to be applied to the second category of pavements is outlined first. The reader will realize that it can readily be extended to the first and third categories as well.

Consider the attributes of the second category of pavements given by equation (89). For constant PCR and deflection, the utility will increase monotonically with ADT, as pavements with higher traffic volumes need quicker consideration. This continuous function can be mathematically approximated by a polynomial :

$$u = \sum_{i=1}^{n} a_i \, (ADT)^i + \lambda_1 \qquad (127)$$

where $\lambda_1 = f1$ (PCR , deflection) and n is the degree of the polynomial.

Similarly, the utility will monotonically increase with increasing deflection and decrease with increasing PCR. Consequent to this argument, equations (128) and (129) can also be deduced for utilities :

$$u = \sum_{i=1}^{n} b_i \, (\text{def.})^i + \lambda_2 \qquad (128)$$

where $\lambda_2 = f2 \, (\text{ADT} , \text{PCR})$

$$u = \sum_{i=1}^{n} c_i \, (\frac{1}{\text{PCR}})^i + \lambda_3 \qquad (129)$$

where $\lambda_3 = f3 \, (\text{def.} , \text{ADT})$

Then, a general equation that accomodates equations (127), (128) and (129) will be :

$$u = \sum_{i=0}^{n} a_i \, (\text{ADT})^i + b_i \, (\text{def.})^i + c_i \, (\text{PCR})^{-i} \qquad (130)$$

This can be rewritten in a more concise form as :

$$u = A_i^T \cdot a_i \qquad (131)$$

where A_i^T is the 3n+1 row vector $[\text{ADT}^i, \text{def.}^i, \text{PCR}^{-i}]$, and a_i is the 1 X 3n+1 column vector $[a_i , b_i , c_i]$.

Equation (131) can be repeatedly applied for 3n+1 known utilities for given attribute values to obtain :

$$u^* = [A^*] \cdot a_i \qquad (132)$$

where u^* is the known 3n+1 X 1 utility vector containing the utilities assigned by an expert for the corresponding attribute values contained in the 3n+1 X 3n+1 matrix $[A^*]$.

On inverting equation (132) :

$$a_i = [A^*]^{-1} \cdot u^* \tag{133}$$

and substituting in equation (131) :

$$u = A_i^T \cdot [A^*]^{-1} \cdot u^* \tag{134}$$

Equation (134) can be rewritten as :

$$u = A_i^T \cdot [B^*] \tag{135}$$

where $$[B^*] = [A^*]^{-1} \cdot u^* \tag{136}$$

By storing the matrix $[B^*]$ for each expert, it is possible to find each expert's concept of utility corresponding to any given 3-tuple. The individual utilities assigned by each expert for the same attribute 3-tuple, can be combined in a manner resembling the formation of acceptable performance levels (see the section on the acceptable serviceability index). This results in a fuzzy utility (different beliefs in different values) for the particular attribute tuple.

However, from the point of view of uncertainty inherent in experts' opinion of utilities, it is more appropriate to ask for a utility interval for a certain 3-tuple, rather than a crisp utility. Equation (134) can then be modified to :

$$\delta u = A_i^T \cdot [A^*]^{-1} \cdot \delta u^* \tag{137}$$

where δu^* contains the utility intervals assigned by each expert to the 3n+1 known 3-tuples in $[A^*]$. The easiest way to derive equation (137) is to consider the product $A_i^T.[A^*]^{-1}$ in equation (134) as a vector A^{**}. Assigning utility intervals for a given set of attribute values, produces a utility interval for the

desired set of attribute values, which can be obtained by differentiating equation (134). Because the elements of A^{**} depend only on the attribute values, the differentiation does not affect A^{**}, thus resulting in equation (137). This modified equation can be programmed in a form similar to equations (135) and (136), to store the expert system in a matrix $[\delta B^{*}]$ similar to $[B^{*}]$:

$$[\delta B^{*}] = [A^{*}]^{-1} . \delta u^{*} \tag{138}$$

$$\delta u = A_I^{T} . [\delta B^{*}] \tag{139}$$

Using equations (138) and (139), a utility interval reflecting the judgment of each expert can be calculated for any desired 3-tuple. Intervals assigned by all experts for a particular 3-tuple can then be gathered to obtain a fuzzy utility. The number of experts supporting a particular utility value for a given attribute 3-tuple is proportional to the membership of utility in the fuzzy utility corresponding to that attribute 3-tuple. This is similar to the formation of acceptable serviceability and friction levels.

A computer program DM (Appendix H) has been prepared to execute the procedure for creating priorities. The above interpolation technique is used in the subroutine INTERPOL, which feeds the main program with the required utility values.

It should be mentioned that this is only one alternative out of many possible interpolations and that even the basic utility equation (equation (130)) can be expressed in a variety of ways.

Expert opinion of pavement priorities enters the picture in obtaining equation (132). Therefore, the questionnaire must seek their opinions on 3n+1 combinations of (ADT,PCR,deflection). Since only two attributes are relevant to groups 1 and 3, the questionnaires corresponding to those groups must seek the opinions of experts on 2n+1 combinations of (friction number, ADT) and (friction life, PSI life), respectively. This gives the freedom to select the number of utilities to be obtained from the highway personnel, by controlling the degree of the polynomial,n, in equation (130) (accuracy of interpolation). It is interesting to note that the accuracy of interpolation (n),

1. has an upper bound defined by the number of utilities that the experts are expected to assign with the least effort,

2. has a lower bound defined by the number of utilities required to cover broad working ranges of attribute values.

On deciding that the interpolation method is practically more viable, questionnaires were prepared (Nos. 6 and 7) to seek the opinion of highway decision makers on the utilities corresponding to 15 to 25 attribute value combinations (n-tuples) for each group. These are found in the Appendix F. This minimizes their inconvenience. The corresponding n values would be from 6 to 8 thereby introducing polynomials of order between 6 and 8 for each group ; This produces an accurate interpolation.

It was described how a primary classification of pavements according to maintenance needs can be done by comparing the PSI

and the friction number with acceptable serviceability and friction levels. Then, attributes relevant to the prioritization of each category according to maintenance urgency was identified. Finally, a decision making model was set up for for creating maintenance priorities, using these attribute values and an expert knowledge base for priority utilities.

CHAPTER 6 : A NUMERICAL EXAMPLE

In this chapter, a complete problem is solved using the pavement evaluation and management techniques proposed in the earlier chapters. The numerical examples are kept simple so that the reader can check the operations by hand and become familiar with the technique.

Let us assume that three asphalt pavement sections 1,2 and 3 are to be ranked for maintenance. Measured performance and traffic data for the sections are shown in Table 11.

The first step of the evaluation is to establish the maintenance category to which each section belongs by determining each section's PSI value. The PSI-Roadmeter reading relationship was obtained from an ideal set of ratings, using a rating panel divided into only two groups, engineers and others. Each group was further divided into two subgroups, consisting of only two raters. Table 12 shows the PSR data obtained from the rating panel observations.

Let us also assume that the following information has been obtained from experts on rideability studies (see variability of roadmeter reading, Chapter 3).

Table 11. Measured performance and traffic data

Pavement property	section 1	section 2	section 3
Roughness	1950	2050	2150
Deflection (10-3 in.) (at center in the fall)	0.5	1.5	1.0
Distress : (see modified instructions appendix E)			
Cracking	16.0	18.0	24.5
Rutting/ shoving	18.5	20.0	21.0
Patches	10.0	11.0	16.5
Pumping	0.0	0.0	0.0
Edge/joint /shoulder condition	1.0	1.0	2.0
ADT	8000	6500	5000

Table 12. Panel ratings

Engineers				Others				Roadmeter reading
Major		Minor		Major		Minor		
rater 1	rater 2	rater 3	rater 4	rater 5	rater 6	rater 7	rater 8	
3.0	2.9	3.0	2.9	3.2	2.8	2.9	2.8	1400
2.9	2.8	2.8	2.7	3.1	2.7	2.7	2.8	1600
2.7	2.6	2.7	2.7	2.9	2.7	2.5	2.5	1800
2.6	2.5	2.4	2.5	2.7	2.6	2.5	2.5	2000
2.4	2.3	2.3	2.4	2.6	2.5	2.3	2.3	2200
2.3	2.3	2.2	2.1	2.4	2.4	2.1	2.3	2400

PSR

	engineers	others
Group concentration/ dilation indices (i.e. relative perceptiveness)	1.6	0.8

	major	minor
Subgroup weights	0.625	0.375

Uncertainty in individual opinion (PSR) = + 0.25

Roadmeter

Uncertainty in the reading due to irrepeatability = 7.5%
Uncertainty in the reading due to variations in
gas tank level = 2.0%
" " " " variations in
vehicle speed = 2.0%
" " " " variations in
driver characteristics = 2.0%

The above information was actually obtained from engineers working in the respective areas through questionnaires 1 and 2 reproduced in Appendix F. These ranges can be used to form kernels of fuzzification of the Roadmeter reading(Chapter 3).

The computer program ROAD is used to formulate the PSR-RR relationship and obtain a fuzzy PSI for a given Roadmeter reading. The PSI fuzzy sets were obtained for the sample sections using the roadmeter readings in Table 11. The results of the computation are found in Tables G1 and G2 (Appendix G). Figure 32 provides a graphical representation of the results.

The next step is to compare these PSI values with the acceptable and unacceptable serviceability ranges. Let the responses of five highway engineers regarding these ranges be as given in Table 13. The corresponding fuzzy sets (see acceptable serviceability levels, Chapter 5) are shown in Figure 33.

With results of ROAD (Tables G1 and G2) the acceptable and unacceptable indices (Table 14), μ_A and μ_N, are obtained for each section (see equation (87) for the computational details).

According to the criteria in equation (87), all three pavement sections belong to the second category of maintenance, i.e., unacceptably rough. (Remark : the data were chosen to achieve this, so that the example can be pursued further). Deflection and condition surveys should then be scheduled for further pavement evaluation. The data resulting from such surveys are also given in Table 11.

The range of variation of deflection parameters, to be obtained from experts (see questionnaire No. 4, Appendix F), are used to form a kernel of fuzzification and fuzzy conversion factors for deflection readings (see fuzzification of measurements,

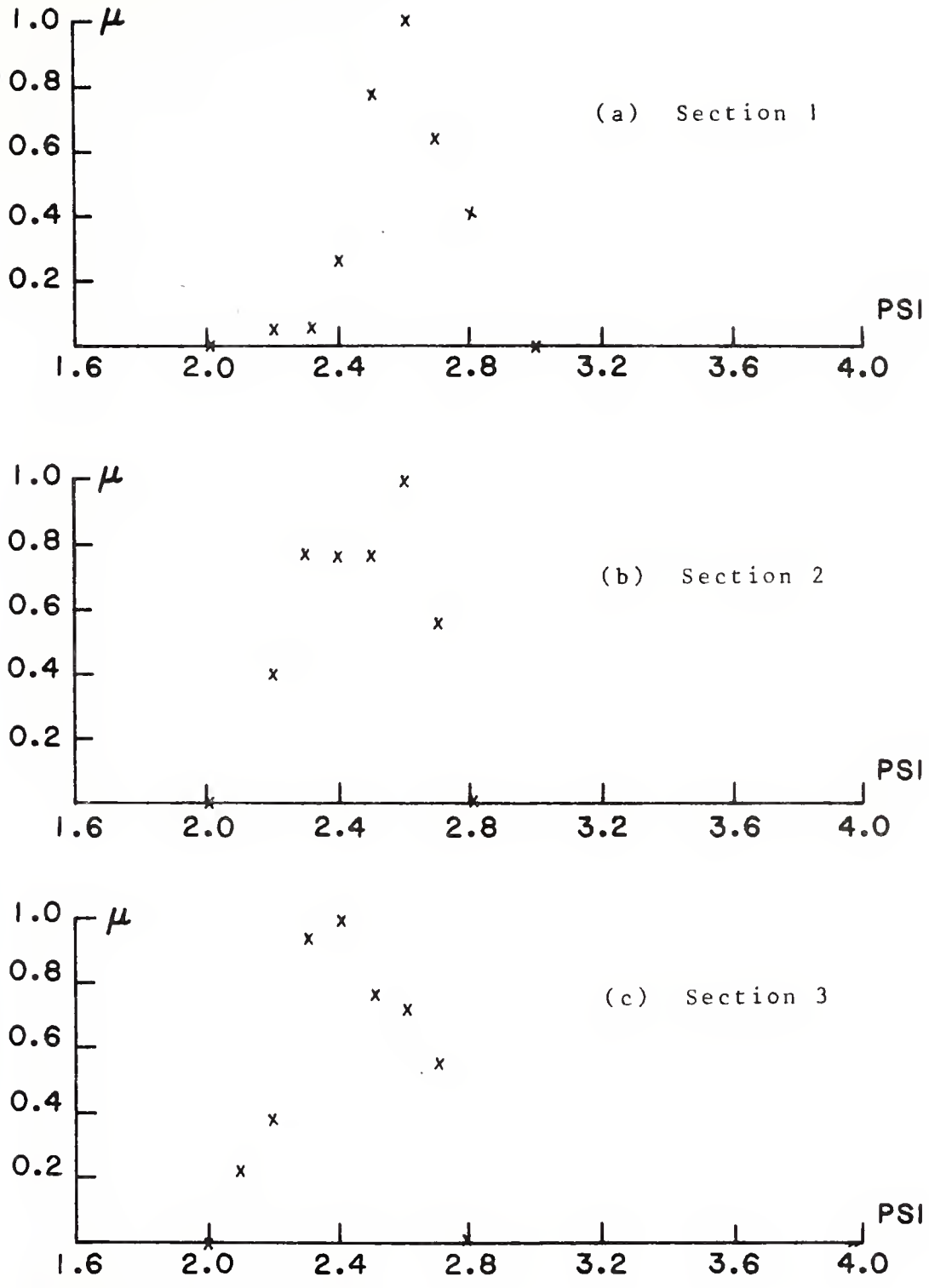


Figure 32. PSI for different Roadmeter readings

Table 13. Acceptable and unacceptable PSI ranges

Expert	Acceptable range	Unacceptable range
1	2.6 - 5.0	0.0 - 2.5
2	2.4 - 5.0	0.0 - 2.4
3	2.7 - 5.0	0.0 - 2.7
4	2.8 - 5.0	0.0 - 2.8
5	2.5 - 5.0	0.0 - 2.5

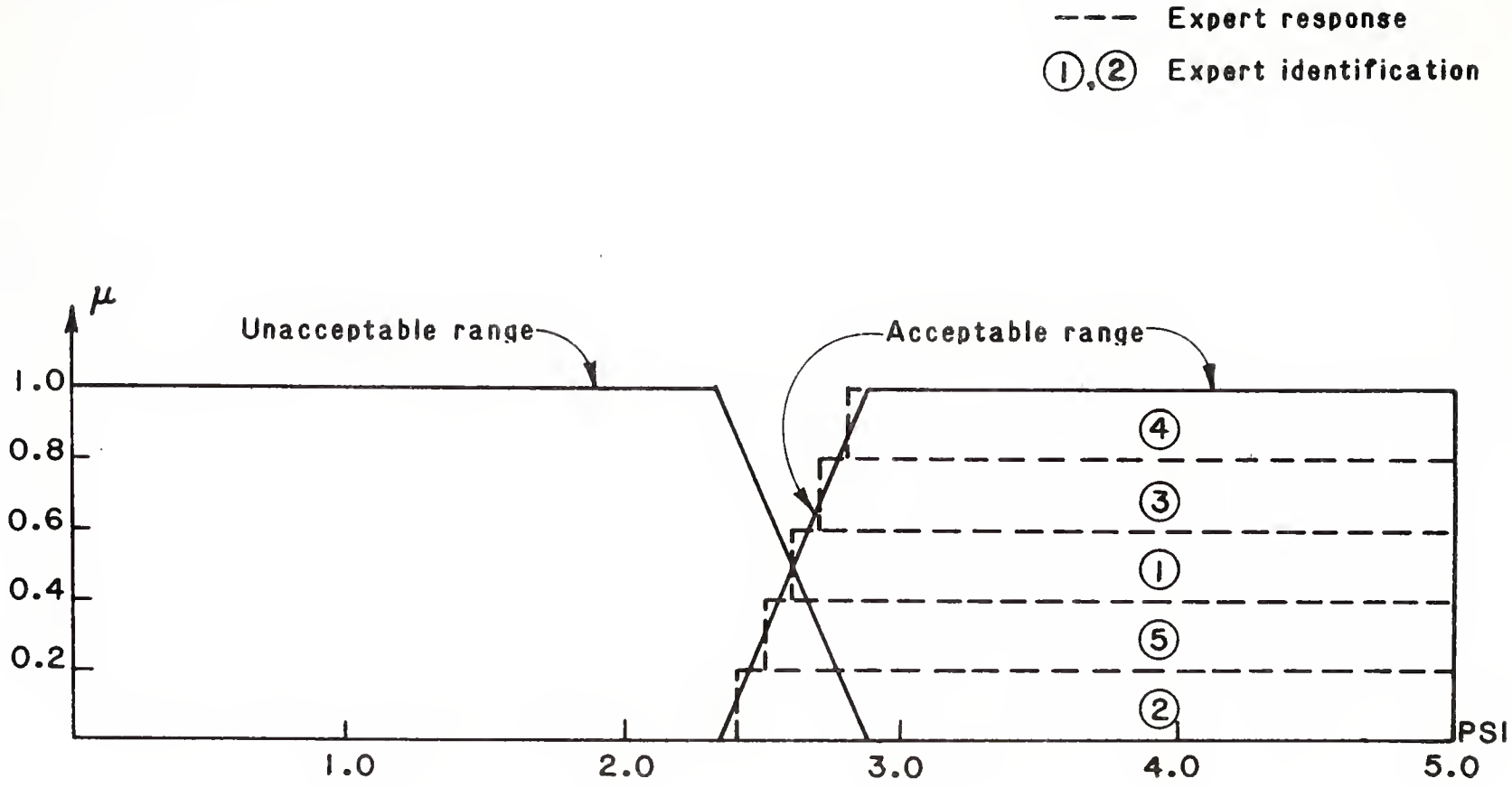


Figure 33. Acceptable serviceability range

Table 14 . μ_A and μ_N indices

Section	μ_A	μ_N
1	0.300	0.363
2	0.231	0.436
3	0.066	0.443

Chapter 4). The deflections in Table 11 have been obtained during the fall season and at the center of the pavement. Such deflections should be standardized to the more critical spring conditions and to the edge of the pavement, using the conversion factors given below :

Deflection

Variation due to irrepeatability	= 10%
Factor to convert fall deflections to those of spring	= 1.5
" " pavement center deflections to those at the edge	= 1.25
Uncertainty associated with the fall-spring conversion factor	= 2%
" " with the center-edge conversion factor	= 5%

The deflection readings are then processed through the "fuzzification" computer program DEF, to give the membership functions of the deflection (Table G3 and Figure 34).

The evaluation data of Table 11 indicate that the modified instructions proposed in the Appendix E were used for the distress survey. In order to incorporate the rating uncertainties in the analysis, the data in Table 15 are assumed (obtained from the distress survey crew itself with questionnaire No. 5, Appendix F). Separate kernels of fuzzification can be formed for the

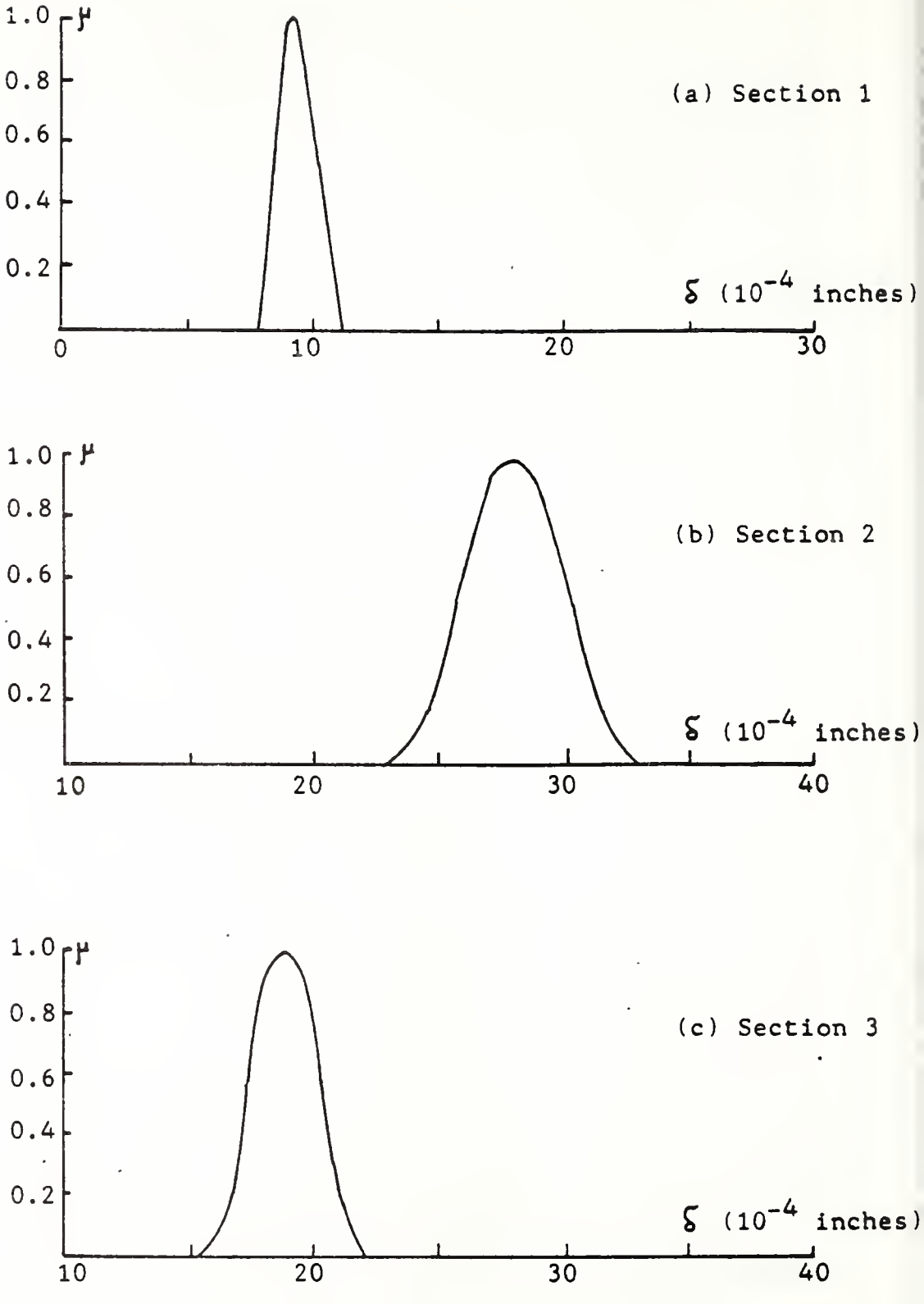


Figure 34. Deflection plots

Table 15. Extents of distress rating uncertainty

Type of distress	uncertainty in extent	uncertainty in severity
Cracking	2%	10%
Rutting and shoving	3%	-
Patches	10%	10%
Pumping	-	-
Edge, joint and shoulder condition	-	-

extent and the severity of the relevant distresses using the information given in Table 15.

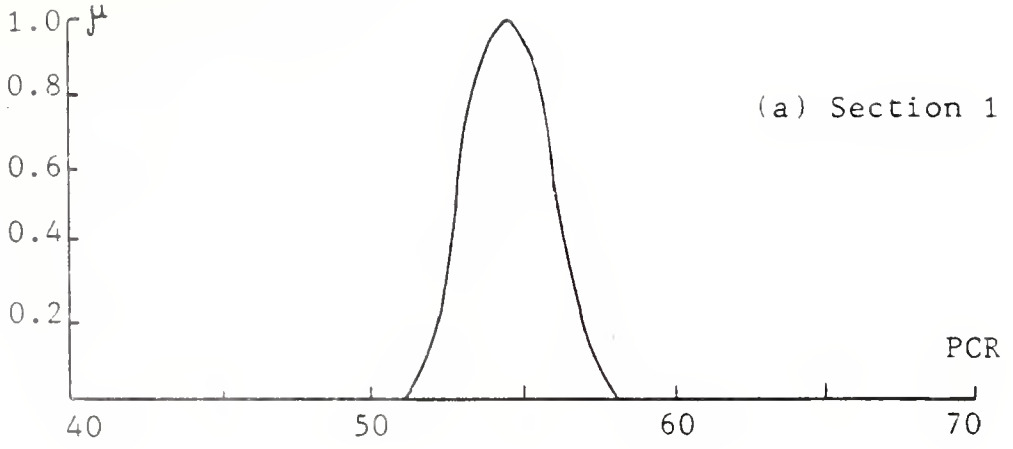
With these data, the fuzzy PCR of each section can be obtained using the computer program DIST. The results of the computations are given in Table G4, and the membership functions are plotted in Figure 35.

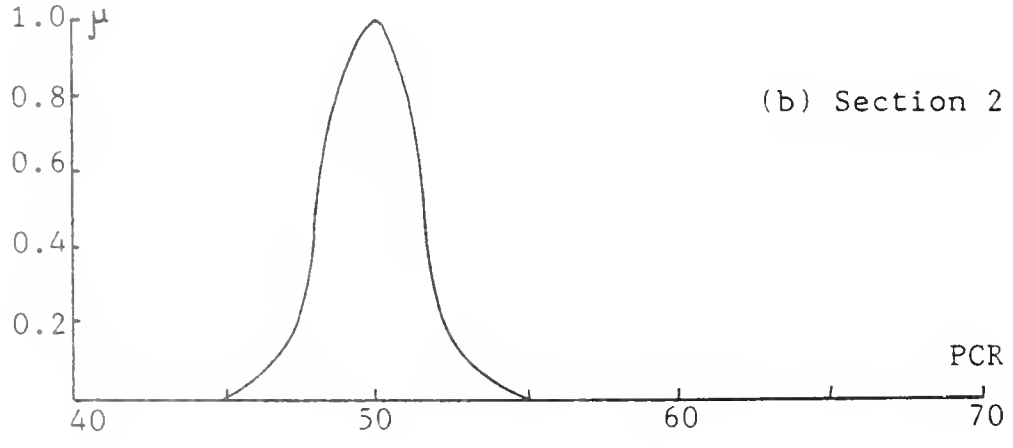
Multi-attribute ranking

Knowing the fuzzy attribute values (Figures 34 and 35, and ADT data in Table 12), the computer program DM can be used to obtain the required ranking for maintenance. In order to do this an expert data base which determines a priority utility for a given combination of attribute values must be created first. As mentioned in Chapter 5, under multi-attribute decision making, a priority scale of 1-10 can be used for this purpose.

Let us assume that the data in Table 16 were obtained from two pavement decision makers.

In constructing the expert knowledge base using the above data, the computer program DM uses the interpolation technique discussed in Chapter 5. Relative priority values (see Equation (123)) obtained for the pavement sections 1,2, and 3 are shown on Table G5 and are reproduced here in Table 17. Therefore, the required ranking would be 1,2 and 3, the highest relative priority value being awarded the most urgency for maintenance.





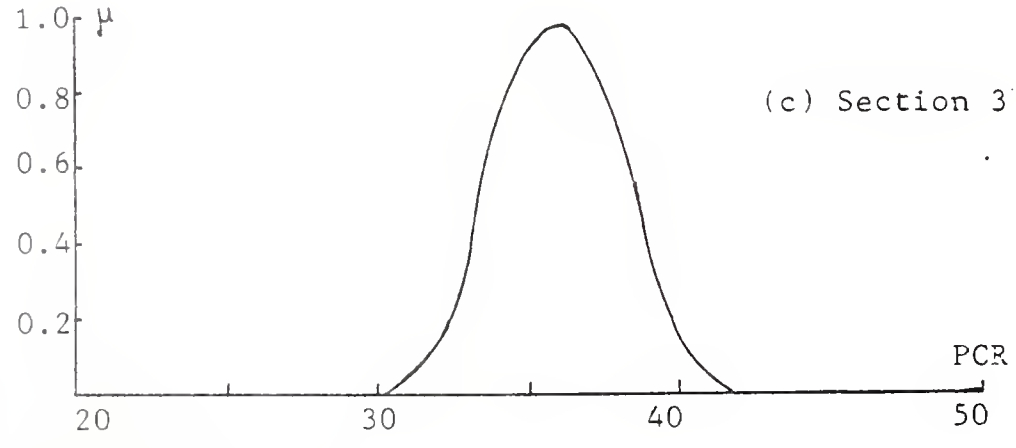


Figure 35. PCR Plots

Table 16 . Expert knowledge base

ADT	defltn.(10^{-3}in.)	PCR	utility	
			expert 1	expert 2
4000	1.0	75.0	4.4-4.6	4.2-4.6
5000	1.5	70.0	5.2-5.3	5.0-5.3
6000	2.0	65.0	6.3-6.5	6.0-6.6
7000	2.0	55.0	7.2-7.45	7.2-7.5
8000	2.0	45.0	8.2-8.4	8.3-8.5
9000	2.2	35.0	9.0-9.15	9.2-9.4
9900	2.5	25.0	9.7-9.9	9.7-9.9

Table 17. Relative priorities

Section	Relative Priority
1	0.85
2	0.50
3	0.16

CHAPTER 7 : SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A methodology for ranking pavement sections in a highway network according to maintenance urgency has been developed with the aid of fuzzy sets mathematics. The new method is formulated within the framework of Indiana's present pavement management system which suffers from imprecise evaluations and subjectivity involved in human decisions. It has been shown that the theory of fuzzy sets can incorporate the system and human uncertainty in the current pavement management procedures by means of membership functions which carry expert judgment into the analysis.

Pavement serviceability indices (PSI) and friction numbers are used in the classification of pavements according to their maintenance needs. Pavement sections with inadequate skid-resistance are ranked for immediate maintenance ; those which do not meet serviceability criteria are also ranked for rehabilitation for current maintenance. Pavements satisfying both serviceability and friction criteria are ranked only for future maintenance.

When the Roadmeter reading (RR) is known, the PSI of a section can be obtained from a pavement serviceability rating (PSR) - RR relationship established for the highway network. In contrast to the conventional PSR which is the average of rideability

ratings assigned by a panel of raters from different backgrounds, the concept of a fuzzy PSR was introduced in the present investigation. Incorporation of uncertainty in the subjectively assigned ratings and of differences in perceptiveness of the raters is certainly an improvement of the fuzzy PSR over the existing method. Rater perceptiveness is introduced through subgroup weights and group concentration and dilation indices. These weights and indices are based on expert judgment. It is also shown that vagueness in expert judgment on concentration and dilation indices lead to a Type 2 fuzzy PSR.

The uncertainty and subjectivity involved in the evaluation of the major pavement properties such as rideabilty, skid-resistance, distress manifestation, and structural adequacy were investigated in detail. Two approaches were created for correlating the Roadmeter reading of a pavement with its PSR. In one approach, the Roadmeter reading is considered as a fuzzy number to account for variability due to changes in climate, gas tank level, driver characteristics and the imprecision of the measuring system. In this approach, the PSR-RR relationship becomes a fuzzy binary relationship. In the second approach, the present notion of random Roadmeter variability is retained and the fuzzy PSR is correlated with the roadmeter reading using fuzzy regression analysis. Fuzzy regression analysis is still in its infancy and the present work is confined to regression coefficients which are symmetric fuzzy sets. It is recommended that this method be improved to include regression coefficients represented by

asymmetric fuzzy sets, in order to provide a better correlation between the PSR and the roadmeter readings.

A major factor affecting the measured friction numbers and deflections is the variation along the test section. Today, this is identified as a random variation and the average result of a specified number of tests is assumed to account for the sectional variation; this number of tests is conventionally found by statistical methods. However, the measured friction numbers have not been statistically correlated with climatic or vehicle speed changes. In this work, it was shown that measured friction numbers can be represented as fuzzy numbers with climatic and vehicle speed changes modelled by kernels of fuzzification to include imprecision in the measured values.

Acceptable serviceability and skid-levels are subjectively determined by experts. A unique demarcation cannot be made between acceptable and unacceptable ranges, because each individual opinion involves uncertainty in addition to being different from the others. It was shown how expert responses on acceptable and unacceptable levels can be directly transformed to fuzzy sets. By using the fuzzy implication operator, a unique criterion was developed for classifying pavements according to maintenance needs, by comparing fuzzy PSI and friction number with the corresponding fuzzy acceptable levels.

Dynaflect tests and condition surveys are conducted on unacceptably rough pavements to evaluate deflection and distress.

Uncertainty due to irrepeatability of deflection measurements is handled by fuzzification of deflection values. A fuzzified deflection, friction number, or Roadmeter reading depict possible values for that measurement, as well as the corresponding degrees of possibility, through memberships. Deflections are considerably affected by seasonal variations and the location of the test across the width of the pavement. The most critical deflections occur at the edge of the pavement during the spring thaw period. However, current practice is to record deflections at the center of the pavements during summer and fall seasons. Factors are presently employed for each type of pavement to convert the measured values to those corresponding to the critical conditions. It is recommended that these crisp factors be replaced by fuzzy conversion factors to account for the uncertainty involved in statistical correlations used in deriving them.

Fuzzy sets are also shown to be useful in distress surveys, since the current methods of visual evaluation of distress certainly introduces human subjectivity into the ratings. The areas where vagueness appears in the distress survey have been identified. Fuzzification of distress ratings leads to the new concept of a fuzzy pavement condition rating (PCR). In the manipulation of PCR, the relative importance of each distress type is presently represented by the mere average of the weights assigned to each type by a panel of experienced engineers in the state. Although the same average weights are used in the fuzzy PCR method, it is possible to further modify this procedure to

incorporate judgment of each member of the panel of engineers on the relative importance of distress types, through fuzzy weights attached to them.

No universal method is available to obtain memberships in the present "state-of-the-art" of fuzzy sets theory. However, various methods have been proposed in the literature as being suitable for different problems. In the present work, a membership function which closely agrees with the implicit analytical definition is used for all symmetric fuzzy sets, while memberships for acceptable performance levels and maintenance priorities are created based on a method resembling exemplification. It is recommended worthwhile to investigate the effect of different membership formulations on the final ranking.

The new ideas proposed in this work of including the perceptiveness of raters and the vagueness inherent in evaluations in the pavement management system were discussed with practicing engineers at meetings arranged regularly. This was also made the first step towards obtaining the subjecive information from the experts. Until now, encouraging response has been obtained from engineers who are working in the specific modules of pavement management system, for the questionnaires covering the extents and the types of variability of Roadmeter reading, friction number, and Dynaflect reading. The author is of the opinion that it would be beneficial for the state to obtain such information from engineers in other states as well.

The Indiana Department of Highways uses the evaluation and traffic data to rank the pavement sections for maintenance and to determine rehabilitation needs. Decision-makers' subjectivity comes into play at the ranking stage as well, even more so when the attribute values or the factors that influence such a decision are numerous. This multi-attribute decision-making process is further complicated by the imprecision in the evaluation parameters. Friction number and the ADT are shown to be attributes of the first category pavements (inadequate skid-resistance), while PCR, deflections and ADT serve as attributes for the second category (unacceptable roughness). Pavement sections which are to be ranked for future rehabilitation (third category) involve a different set of attributes ; the remaining PSI and friction lives. It has been illustrated how service lives can be derived from the performance curves knowing the evaluation results and the acceptable levels.

A new methodology for ranking pavement sections for all three categories was developed using concepts of fuzzy multi-attribute decision-making. The root of this approach is the assumption that a decision-maker considers only the magnitudes of the relevant attributes in assigning a rank for a pavement section. The need then arises for an expert knowledge base which determines a priority value, on a predetermined scale, for a given combination of attribute values. This was achieved through questionnaires , where decision-makers' judgment of the relative priorities are sought on a number of selected combinations of

attribute values relevant to each category. When a number of expert opinions are gathered, the priority values are better represented by fuzzy numbers. To encounter this situation, the method was extended to cover fuzzy priority values as well. Using this data base and by employing an interpolation technique, it is shown how a fuzzy priority value is obtained for any desired combination of attribute values. This results in fuzzy priority values for each pavement section. Then, by defining maximizing priority sets for the pavement sections and by combining them with the corresponding priority sets, a set of crisp ranking indices are extracted. Thus, using evaluation results, traffic data, and expert knowledge a unique criterion for ranking pavement sections has been formulated for implementation of maintenance.

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APPENDICES

APPENDIX A

Fuzzy calculus

Extension principle and composition

The projection of an n-ary fuzzy relation R , on $X_1 \times \ldots \times X_r$ where $(1,\ldots,r)$ is a subseqence of $(1,\ldots\ldots\ldots,n)$, is a relation on $X_1 \times \ldots \times X_r$ and is defined by Zadeh(1975) as:

$$\text{Proj. } R = \int_{x_1,\ldots,x_r} \left[\sup_{x_{r+1},\ldots,x_n} \mu_R(x_1,\ldots,x_n)\right]/(x_1,\ldots,x_r) \qquad (A1)$$

with the supremum taken over all memberships of n-tuples containing any values of $x_{r+1},\ldots\ldots,x_n$, where $(r+1,\ldots\ldots,n)$ is the sequence complementary to $(1,\ldots,r)$.

Conversely, if R is a fuzzy set in $X_1 \times \ldots\ldots \times X_r$, then its cylindrical extension in $X_1 \times \ldots\ldots \times X_n$ is a fuzzy set c(R) on $X_1 \times \ldots \times X_n$ defined by Zadeh (1975) as :

$$c(R) = \int \mu_R(x_1,\ldots,x_r) \,/\, (x_1,\ldots,x_n) \qquad (A2)$$

The composition of two fuzzy relations R and S on $X_1 \times \ldots \times X_r$ and $X_s \times \ldots\ldots \times X_n$ respectively, with $s \leq r$ is expressed by Zadeh (1975) as :

$$R.S = \text{Proj.}[c(R) \cap c(S)] \text{ on } [X_1 \times \ldots \times X_{s-1}] \times [X_{r+1} \times \ldots \times X_n] \qquad (A3)$$

c(R), c(S), and $c(R) \cap c(S)$ are fuzzy sets on the space

$X_1 x \ldots . x X_n$, and the composition of R and S is a fuzzy set on the subspace complementary to the subspace common to R and S, i.e. $X_s x \ldots . x X_r$. This is further illustrated by the discussion of composition in Chapter 2.

The extension principle (equation 19) can be written as :

$$\mu_B(y) = \sup_{\substack{x_i \\ y=f(x_i)}} \min [\mu_{A_1}(x_1) \ldots . \mu_{A_n}(x_n)] \qquad (19)$$

where the supremum is taken over all values of $x_1, \ldots x_n$, satisfying $y = f(x_1, \ldots . . x_n)$.

In the sequel it will be shown that the fuzzy set B in equation (19) can be obtained by the composition of two relations R and S defined in a special manner.

$$\text{if} \quad R = c(A_1) \ , \ldots . . , \ c(A_n), \qquad (A4)$$

then

$$\mu_R(x_1, \ldots, x_n) = \min [\mu_{A_1}(x_1), .., \mu_{A_n}(x_n)], \qquad (A5)$$

and

$$c(R) = \int \mu_R(x_1, .., x_n)/(x_1, .., x_n, y) \qquad (A6)$$

where c(R) is the cylindrical extension of R on $X_1 x \ldots . x X_n x$ Y.

Let us define another fuzzy relation S, in $X_1 x \ldots . x X_n x\ Y$ with a membership function :

$$\mu_S(x_1, \ldots . x_n, y) = 1.0 \quad \text{for} \quad y = f(x_1, \ldots, x_n).$$

$$c(S) = U\ \mu_S(x_1, .., x_n, y)/(x_1, \ldots, x_n, y) \tag{A7}$$

From equations (A6) and (A7) it follows that :

$$\mu_{c(R) \cap c(S)} = \mu_R(x_1, \ldots, x_n) \tag{A8}$$

Further, equations (A1), (A3) and (A8) give :

$$\mu_{R.S}(y) = \sup_{\substack{x_i \\ y=f(x_i)}} \mu_R(x_1, \ldots . , x_n) \tag{A9}$$

It is also noticed from equations (A5) and (19) that :

$$\mu_B(y) = \sup_{\substack{x_i \\ y=f(x_i)}} \mu_R(x_1, \ldots . , x_n) \tag{A10}$$

Equations (A9) and (A10) produce the desired result as :

$$B = R.S$$

Agreement of π curves with implicit analytical definition of memberships

Equations (24) and (25) (in Chapter 3) can be combined to form :

$$d/dr\ [\mu_c(r)] = k\ \mu_c(r)\ .\ [1 - \mu_c(r)] \qquad (A11)$$

where k is the constant of proportionality. Thus,

$$k\ r = \int \frac{d\mu_c(r)}{\mu_c(r)(1 - \mu_c(r))} \qquad (A12)$$

On integrating, equation (26) is obtained :

$$\mu_c(r) = \frac{e^{kr+c}}{1 + e^{kr+c}} \qquad (26)$$

where c is the constant of integration.

The following table contains values of memberships obtained by using equations (26) and (29) with the respective boundary conditions, ($\delta = 0.01$) for the corresponding k values where :

$$k = \frac{\gamma - x}{\beta}$$

Figure A1 is a plot of these results.

Table A1. Comparison of π curves and exponential curves

k	membership (from π curve)	membership (from. exponential curve)
0.0	1.00	0.99
0.1	0.98	0.98
0.2	0.92	0.94
0.3	0.82	0.86
0.4	0.68	0.71
0.5	0.50	0.50
0.6	0.32	0.29
0.7	0.18	0.14
0.8	0.08	0.06
0.9	0.02	0.03
1.0	0.00	0.01

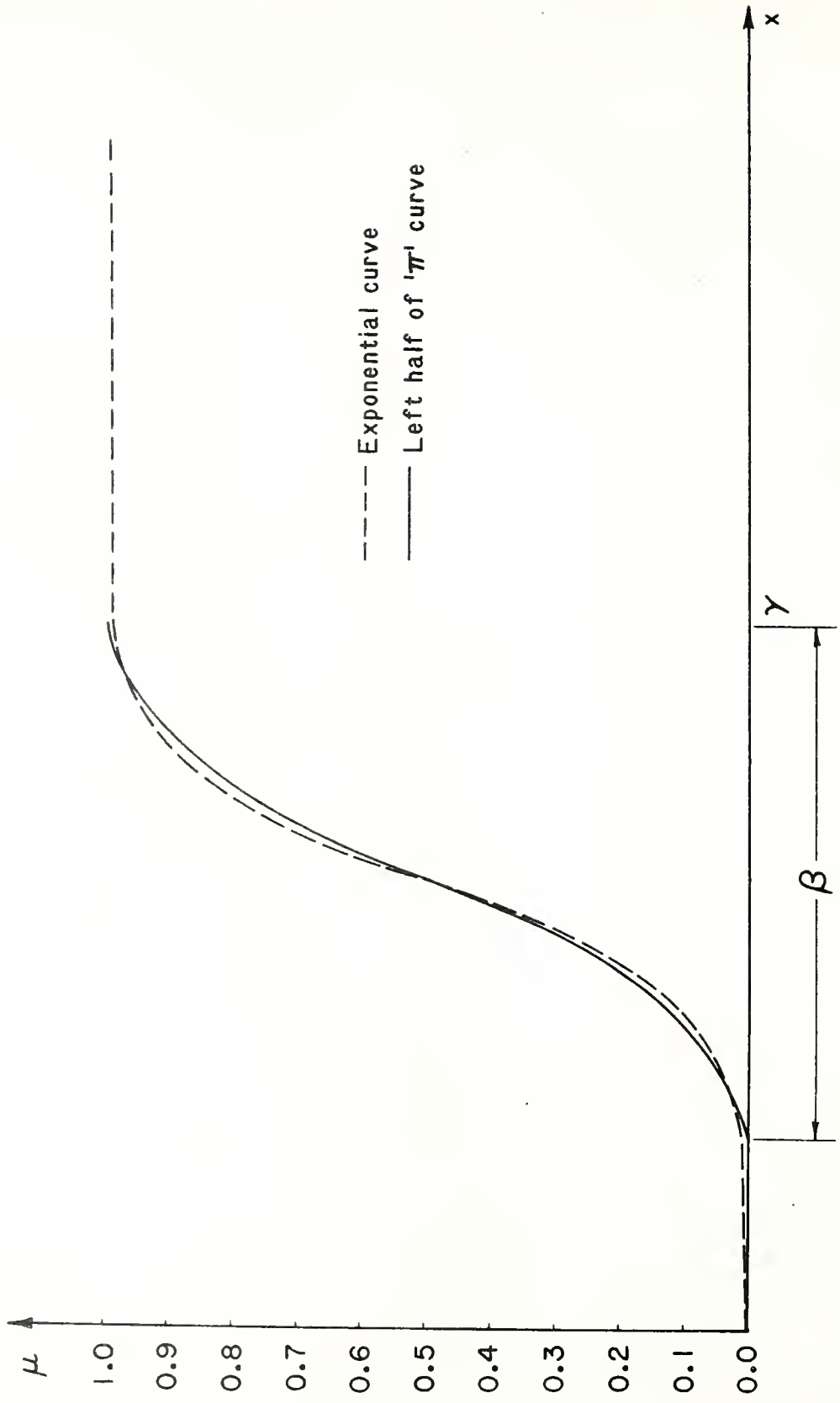

Figure A1. Comparison of exponential and 'π' curves

APPENDIX B

APPENDIX B

Type 2 fuzzy sets

The exponential and scalar multiplication of a fuzzy number α (equation (45), Chapter 3) are given by, respectively (Dubois and Prade, 1980):

$$e^{\alpha} = \bigcup_j f(\mathrm{Ln} y_j)/y_j \qquad \text{for } y_j > 0 \tag{B.1}$$

$$\lambda\alpha = \bigcup_j f(\frac{y_j}{\lambda})/y_j \tag{B.2}$$

where λ is an arbitrary scalar and $f(.)$ is the membership function of α. Equation (B.1) and (B.2) can also be written as :

$$e^{\alpha} = \bigcup_j f(y_j)/e^{y_j} \tag{B.3}$$

$$\lambda\alpha = \bigcup_j f(y_j)/\lambda y_j \tag{B.4}$$

Applying equation (B.3) to the fuzzy number $\lambda\alpha$ yields:

$$e^{\lambda\alpha} = \bigcup_j f(y_j)/e^{y_j{}^{\lambda}} \tag{B.5}$$

Since the membership value $\mu(x_i)$ (equation (44), Chapter 3) is a scalar, its Naperian logarithm is also a scalar which can be substituted for λ in equation (B.5):

$$e^{\mathrm{Ln}\mu(x_i)\alpha} = \bigcup_j f(y_j)/e^{y_j \mathrm{Ln}\mu(x_i)} \tag{B.6}$$

this can be simplified to :

$$\mu(x_i)^{\alpha} = \underset{j}{U}\ f(y_j)/[\mu(x_i)]^{y_j} \qquad (B.7)$$

Substituting this expression in equation (44) :

$$A^{\alpha} = \underset{i}{U}\ (\underset{j}{U}\ f\ (y_j)\ /[\mu(x_i)]^{y_j}\ )/x_i \qquad (B.8)$$

Hence equations (46a) and (46b) are obtained.

APPENDIX C

APPENDIX C

Fuzzy linear regression

Since the proof of the result shown in equations (55) and (56) is rather complicated, it is first given for one random variable, (equations (66) and (67)). Then, it is shown how the proof can be easily extended to the case of n variables.

To show that fuzzy PSI set is a π curve when A_1 and A_2 are assumed to be π curves, assume that the latter are shown by Figure C1. It follows from the definition of π curves [equation (29a)] that :

$$\mu_{A_i}(a_i) = 2\left[\frac{a_i - (\gamma_i - \beta_i)}{\beta_i}\right]^2 \quad \text{for} \quad \gamma_i - \frac{\beta_i}{2} \geq a_i \geq \gamma_i - \beta_i \qquad (C1)$$

$$= 1 - 2\left[\frac{a_i - \gamma_i}{\beta_i}\right]^2 \quad \text{for} \quad \gamma_i \geq a_i \geq \gamma_i - \frac{\beta_i}{2} \qquad (C2)$$

along with the condition of symmetry with respect to γ_i.

By introducing :

$$a_i = \gamma_i \pm k\beta_i$$

equation (C1) gives :

$$\mu_{A_i}(a_i) = f(k), \quad \text{for all } a_i \qquad (C3)$$

which can be identified as a property of π curves (Figure C1).

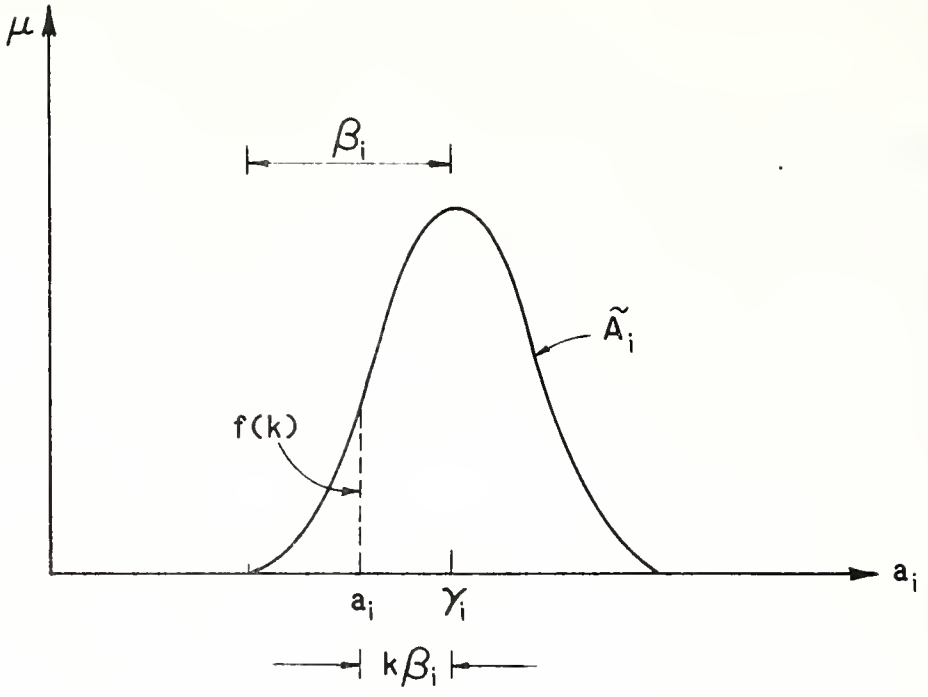

Figure C1. Fuzzy set $\tilde{A}_i$

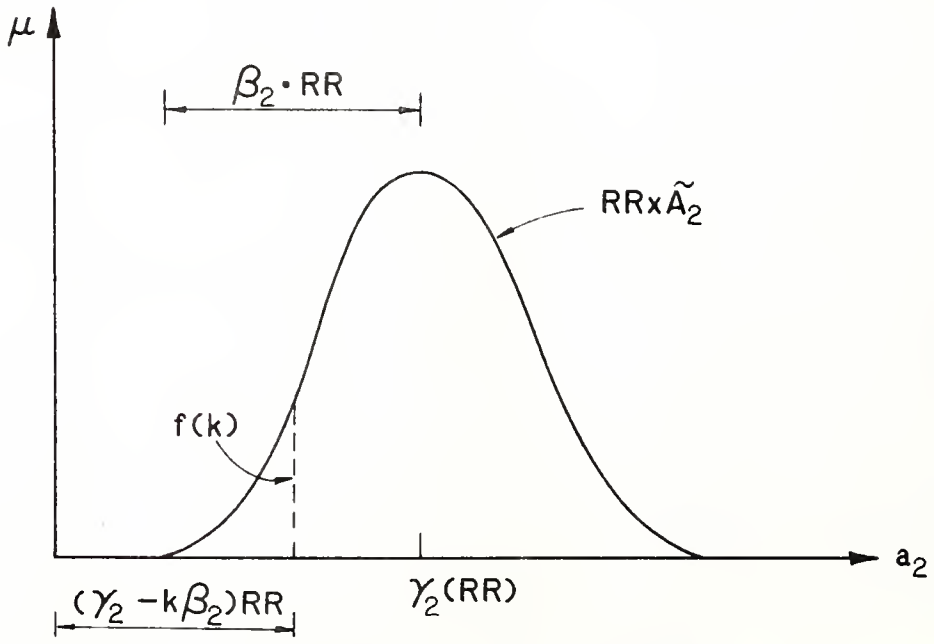

Figure C2. Fuzzy set RR x $\tilde{A}_2$

From the extension principle, and since RR is a random variable, A_2 x RR is represented by the π curve in Figure C2 :

$$\mu_{RRxA_2}(a'_2) = \max_{a'_2=RRxa_2} [\mu(a_2)] \qquad (C4)$$

Once again the extension principle is used to obtain the membership function for PSI :

$$\mu_{\Psi}(y) = \sup_{y=a_1 + a'_2} \min[\mu_{RRxA_2}(a'_2),\ \mu_{A_1}(a_1)] \qquad (C5)$$

Consider a value of y given by :

$$y = (\gamma_1 - k\beta_1) + (\gamma_2 - k\beta_2)RR$$

There are 3 ways to obtain this value :

(1) $a_1 = \gamma_1 - k\beta_1$ and $a'_2 = \gamma_2\ RR - k^{\cdot}RR^{\cdot}\beta_2$

(2) $a_1 < \gamma_1 - k\beta_1$ and $a'_2 > \gamma_2{}^{\cdot}RR - k^{\cdot}RR^{\cdot}\beta_2$

(3) $a_1 > \gamma_1 - k\beta_1$ and $a_2 < \gamma_2{}^{\cdot}RR - k^{\cdot}RR^{\cdot}\beta_2$

When all these possibilities are treated according to equation (C5), $\mu_{PS1}(y)$ is given by the critical condition corresponding to the first possibility:

$$\mu_{PS1}[y] = f(k) \qquad (C6)$$

when $y = (\gamma_1 - k\beta_1) + (\gamma_2 - k\beta_2)RR$

or $y = (\gamma_1 + \gamma_2 \cdot RR) - k(\beta_1 + \beta_2 \cdot RR)$

Hence, using the property of π curves obtained in equation (C3), it can be deduced that the PSI has a membership function described by a π curve with :

$$\gamma = \gamma_1 + \gamma_2 \cdot RR \tag{66}$$

and $$\beta = \beta_1 + \beta_2 \cdot RR \tag{67}$$

In general, this shows that if A_1 and A_2 are described by $\pi(\gamma_1, \beta_1)$ and $\pi(\gamma_2, \beta_2)$ repectively, then Y_1 in :

$$Y_1 = A_1 x_1 + A_2 x_2 \tag{C7}$$

is also described by a π curve.

By induction, it follows that Y_2 in :

$$Y_2 = Y_1 + A_3 x_3 \tag{C8}$$

is also described by a π curve since A_3 and Y_1 are π curves. By repeating this up to A_n, it follows that Y_n in :

$$\begin{aligned} Y_n &= Y_{n-1} + A_n x_n \\ \text{or} \quad &= A_1 x_1 + \ldots\ldots + A_n x_n \end{aligned} \tag{C9}$$

is described by a π curve.

In the same manner, equations (55) and (56) can be derived starting from equations (66) and (67).

APPENDIX D

APPENDIX D

Some useful numerical techniques

Fuzzification

Case (i) Combination of two discrete kernels :

$$K_1(y^*) = \underset{i}{U}\ \mu_i/y_i \qquad y^* - \alpha \leq y_i \leq y^* + \alpha \tag{D1}$$

$$K_2(y^*) = \underset{j}{U}\ \mu_j/y_j \qquad y^* - \theta \leq y_j < y^* + \theta \tag{D2}$$

From the properties of a kernel described in Chapter 3 (see section on fuzzification) the composite kernel is given by :

$$K(x) = K_1(x) \,.K_2(x) \tag{48a}$$

$$K = \underset{y_i=y^*-\alpha}{\overset{y_i=y^*+\alpha}{U}} \mu_i \cdot \underset{y_j=y_i-\theta}{\overset{y_j=y_i+\theta}{U}} \mu_j/y_j \tag{D3}$$

When K operates on a number, case (iv) below may be used for the manipulation.

Case (ii) Combination of a continuous kernel and a discrete kernel :

If the discrete kernel is given by :

$$K_1(y^*) = \underset{i}{U}\ \mu_i/y_i \qquad y^* - \alpha \leq y_i \leq y^* + \alpha \tag{D4}$$

and the continuous kernel is given by :

$$K_2(y^*) = \int_{\theta} \mu/y \qquad y^* - \theta \leq y \leq y^* + \theta \tag{D5}$$

Then from equation (48a) and the properties of a kernel :

$$K = \int_{y=y^*-\theta}^{y=y^*+\theta} \; \bigcup_{y_i=y-\alpha}^{y_i=y+\alpha} \mu \cdot \mu_i / y_i \tag{D6}$$

This will be a continuous fuzzy set centered around y^* , with a half-width $\alpha + \theta$.

When K operates on a number, techniques in case (v) may be used for manipulation.

Case (iii). Combination of two continuous kernels :

$$K_1(y^*) = \int_{\alpha} \mu/y \qquad y^* - \alpha \leq y \leq y^* + \alpha \tag{D7}$$

$$K_2(y^*) = \int_{\theta} \mu'/y' \qquad y^* - \theta \leq y' \leq y^* + \theta \tag{D8}$$

From the properties of kernels and equation (48a) :

$$K(y^*) = \int_{y'=y^*-\theta}^{y'=y^*+\theta} \; \int_{y=y'-\alpha}^{y=y'+\alpha} \mu \cdot \mu'/y \tag{D9}$$

This, is a continuous fuzzy operator centered around y^*. It can operate on a crisp number using the techniques in Case(v) below.

Case (iv). Discrete kernel operating on a crisp number :

Let a discrete kernel be defined by :

$$K(y^*) = \underset{i}{U}\ \mu_K(y_i)/y_i \qquad y^* - \theta \leq y_i \leq y^* + \theta \tag{D10}$$

Operating this kernel on a crisp number x results in :

$$F = K\ x$$

$$F = [\underset{i}{U}\ \mu_K(y_i)/y_i].[1.0/x]$$

$$F = \underset{i}{U}\ \mu_K(y_i)/y_i \qquad x - \theta \leq y_i \leq x + \theta \tag{D11}$$

Case (v) Continuous kernel operating on a crisp number :

$$\text{Let } K(y^*) = \int_{\theta} \mu_K(y)/y \qquad y^* - \theta \leq y \leq y^* + \theta \tag{D12}$$

Then,

$$F = [\int_{\theta} \mu_K(y)/y\ ].[\ 1.0/x]$$

$$F = \int_{\theta} \mu_K(y)/y \qquad x - \theta \leq y \leq x + \theta \tag{D13}$$

This is a fuzzy set defined by the kernel itself but centered around x.

Use of fuzzy factors

The following is a continuation of the discussion on fuzzy factors in Chapter 4.

Case (iii)a. Multiplication of a continuous fuzzy set by a discrete fuzzy factor :

$$f = \bigcup_i \mu_i / y_i \qquad y^* - \theta \leq y_i \leq y^* + \theta \tag{D14}$$

$$\delta = \int_\alpha \mu / \delta \qquad \delta^* - \alpha \leq \delta \leq \delta^* + \alpha \tag{D15}$$

$$F = f . \delta$$

$$= [\bigcup_i \mu_i / y_i][\int_\alpha \mu / \delta]$$

From the extension principle :

$$F = \bigcup_i \int_\alpha \mu \quad \mu_i / y_i \, \delta$$

if $y_i \delta = x_k$, then

$$F = \bigcup_i \int_\alpha \mu \wedge \mu_i / x_k \tag{D16}$$

where $(y^* - \theta)(\delta^* - \alpha) \leq x_k \leq (y^* + \theta)(\delta^* + \alpha)$

Case (iii)b. Similarly, a continuous fuzzy factor can be multiplied by a discrete fuzzy set. It should be noted that in both cases (iii)a, and (iii)b, a continuous fuzzy set will result.

Case (iv). Multiplication of a continuous fuzzy set by a continuous fuzzy factor :

$$f = \int_{\theta} \mu/y \qquad y^* - \theta \leq y \leq y^* + \theta \tag{D17}$$

$$\delta = \int_{\alpha} \mu'/\delta \qquad \delta^* - \alpha \leq \delta \leq \delta^* + \alpha \tag{D18}$$

$$F = f.\delta$$

$$= [\int_{\theta} \mu/y\ ][\int_{\alpha} \mu'/\delta]$$

if $x = y.\ \delta$, then

$$F = \int_{\alpha}\int_{\theta} \mu \wedge \mu'/x \tag{D19}$$

where $(y^* - \theta)(\delta^* - \alpha) \leq x \leq (y^* + \theta)(\delta^* + \alpha)$

APPENDIX E . Distress survey instructions

FLEXIBLE PAVEMENT RATING

DEFECT	RATING SCALE	RATING CRITERIA BY PAVEMENT TYPE	RATING	DEFECT DESCRIPTION
Transverse Cracks	0 5	0 4 hairline cracks = 0 1 (rating) 5 10 hairline cracks = 2 3 (rating) more than 10 hairline cracks = 4 5 (rating)	———	Cracks approximately at right angles to centerline Rate the extent and severity of cracking Extent is rated relative to number of cracks in 200 foot section Severity is rated from hairline cracks (less than 1/8 inch) to severe cracks (greater than ¼ inch) Sealed cracks are rated as hairline unless reopened Severe cracks should be rated higher than hairline cracks Cracks less than 18 inches in length are not considered
Longitudinal	0 5	0 50 feet of hairline cracking = 0 1 (rating) 51 100 feet of hairline cracking = 4 5 (rating)	———	Cracks approximately parallel to the pavement centerline Rate the extent and severity of cracking Extent is rated relative to total length of cracks in 200 foot section Individual cracks are added to obtain estimated total length of cracking Severity is rated as the average width of crack from hairline (less than 1/8 inch) to severe (greater than ¼ inch) Sealed cracks rated as hairline unless opened Severe cracks should be rated higher than hairline cracks Cracks less than 18 inches in length are not considered
Alligator Cracks	0 10	0% = 0 (rating) thru 100% = 10 (rating)	———	Interconnecting cracks forming the typical patterns resembling an alligator's skin Sealed cracks are rated the same as open cracks Extent is rated relative to percentage of pavement surface area in 200 foot section denoting alligator cracking Severity is rated from the initial appearance of fatigue cracks in the wheel paths to other pattern type fatigue cracks generally outside the wheel paths A higher rating should be assigned to severe alligator cracking
Block Cracking	0 5	0% = 0 (rating) thru 100% = 10 (rating)	———	Interconnecting cracks forming series of large polygons usually having sharp angles at corners Rate extent and severity of cracking Extent is rated relative to percentage of pavement surface area in 200 foot section cracked Severity is rated from initial cracking (less than 1/16 inch) to severe cracking (greater than ½ inch) A higher rating should be assigned to severe block cracking
Rutting	0 10	Average rutting ¼ ½" = 0 2 (rating) Average rutting ½ ¾ = 3 5 (rating) Average rutting over ¾ = 6 10 (rating)	———	A rut is a longitudinal surface depression in the wheel paths at least 20 feet long Rate the severity of depth of rutting in 200 foot section as outlined under Rating Criteria To obtain average rutting at least 10 readings should be taken throughout the rutting area and averaged Ruts less than ½ inch in depth are not rated
Shoving or Pushing	0 10	0% = 0 (rating) thru 100% = 10 (rating)	———	Shoving or pushing of pavement layer due to insufficient bond and stability Typically found at intersections resulting from vehicle breaking action Extent of shoving or pushing is rated relative to the percentage of surface area in 200 foot section denoting defect No degrees of severity are defined
Patches	0 10	0% = 0 (rating) thru 100% = 10 (rating)	———	Patches are temporary or permanent corrections to damaged pavements Rate extent and severity of patching Rate extent relative to the percentage of pavement surface area in 200 foot section which is patched Rate severity relative to the condition of the patches from good condition performing satisfactorily to poor condition ride quality is significantly affected A higher rating should be assigned to poor condition patches
Excessive Asphalt	0 10	0% = 0 (rating) thru 100% = 10 (rating)	———	Excessive (bleeding) asphalt is film of bituminous material on the pavement surface which creates a shiny glass like reflecting surface that usually becomes quite sticky Extent of bleeding is rated relative to percentage of pavement surface area in 200 foot section denoting defect No degrees of severity are defined
Pumping and Water Bleeding	0 10	If no pumping observed rate as 0, If any amount of pumping is observed rate as a 10	———	Pumping is the ejection of water and fine materials under pressure through cracks under moving loads Water bleeding occurs where water seeps slowly out of cracks in the pavement surfaces If pumping or water bleeding exists anywhere in the sample unit it is counted as occurring
Corrugations	0-5	0% = 0 (rating) thru 100% = 10 (rating)	———	Corrugation is a form of plastic movement typified by ripples across the asphalt pavement surface Rate the extent and severity of corrugations Extent is rated relative to percentage of pavement surface area in 200 foot section denoting corrugations Severity is rated from light corrugations causing some vibration of the vehicle but creates no discomfort, to severe corrugations causing excessive vibration of the vehicle which creates substantial discomfort and/or a safety hazard, and/or vehicle damage requiring a reduction in speed for safety A higher rating should be assigned to severe corrugations
Raveling	0 5	0% = 0 (rating) thru 100% = 5 (rating)	———	Raveling is the progressive disintegration of the surface downward Rate the extent and severity of raveling Extent is rated relative to the percentage of 200 foot section raveled Severity is from moderately rough and pitted (largely fine agg) to severely rough and pitted (includes coarse agg) A higher rating should be assigned to severe raveling
Polished Aggregate	0 5	0% = 0 (rating) thru 100% = 5 (rating)	———	Polished aggregate is present when close examination of a pavement reveals that the portion of aggregate extending above the asphalt is either very small or there are no rough or angular aggregate particles to provide good skid resistance The extent of polished aggregate is rated relative to the percentage of surface area in 200 foot section denoting and polishing No degrees of severity are defined
Overall Riding Quality	0 10	Roughness No 1400-1750 = 0 3 (rating) Roughness No 1751 2380 = 4 7 (rating) Roughness No 2381 5800 = 8 10 (rating)	———	Roughness numbers appears on reverse side and is used to assign rating

Sum of Defect Ratings ———

Condition Rating = 100 Sum of Defect Ratings

= 100 ———

Condition Rating = [ ]

Comments ———

RIGID PAVEMENT RATING

RATING CRITERIAL BY PAVEMENT TYPE

DEFECT	RATING SCALE	20 ft Jointed PCP or RCP	40 foot Jointed RCP	CRC	RATING	DEFECT DESCRIPTION
Transverse Cracks	0-10	0-7 hairline cracks = 0-1 (rating) 3-5 hairline cracks = 2-3 (rating) more than 5 hairline cracks = 4-10 (rating)	0-2 hairline cracks = 0-1 (rating) 3-5 hairline cracks = 2-3 (rating) more than 5 hairline cracks = 4-10 (rating)	0-70 hairline cracks = 0-4 (rating) 70-100 hairline cracks = 2-3 (rating) more than 100 hairline cracks = 4-10 (rating)		Cracks (not including joints) approximately at right angles to Centerline. Rate the extend severity of cracking. Extent is rated relative to number of cracks in 200 foot section. Severity is rated from hairline cracks (less than 1/8 inch) to severe cracks (greater than ¼ inch). Sealed cracks are rated as hairline cracks unless reopened. Severe cracks should be rated higher than hairline cracks. Cracks less than 18 inches in length are not considered.
Longitudinal Cracks	0-10	0-80 feet of hairline cracking = 0-1 (rating) 81-100 feet of hairline cracking = 2-3 (rating) over 100 feet of hairline cracking = 4-10 (rating)				Cracks (not including joints) approximately parallel to the pavement centerline. Rate the extent and severity of cracking. Extent is rated relative to total length of cracks in 200 foot section. Individual cracks are added to obtain estimated total length of cracking. Severity is rated as the average width of crack from hairline (less than 1/8 inch) to severe (greater than ¼ inch). Sealed cracks rated as hairline unless reopened. Severe cracks should be rated higher than hairline cracks. Cracks less than 18 inches in length are not considered.
D Cracking	0-10	0% = 0 (rating) through 100% = 10 (rating)				"D" cracking is a series of closely spaced crescent shaped hairline cracks that appear at the PCC pavement slab surface adjacent and roughly parallel to joints, cracks and the free edges of a pavement slab. Rate extent and severity of D-cracking. Extent is rated relative to the percentage of joints and cracks in 200 foot section denoting defect. Severity is rated from light pattern of closely spaced line cracks that has developed near joints, cracks, and/or free edges, however, width of affected area is generally less than 12 inches wide at the center of the lane: to severe, a high level of spalling at joints/cracks exist and considerable material is loose in affected area. Also width of affected area is generally greater than 12 inches wide at the center of the lane. A higher rating should be assigned to severe D-cracking.
Crack Spalling	0-10	0% = 0 (rating) through 100% = 10 (rating)				Breakdown or disintegration of edges of cracks, resulting in the loss of concrete and progressive widening of the cracks. Rate extent and severity of spalling. Extent of crack spalling is rated relative to the percentage of cracks in 200 foot section indicating spalling. Severity is rated from negligible spalling, overall average width less than or equal to ¼ inch, to severe spalling, filled or non-filled cracks, having an overall average spalled width greater than or equal to 1½ inches. A higher rating should be assigned to severe crack spalling.
Faulted Transverse Joints or Cracks	0-10	0 Faults = (0 rating) 1 Fault = 11 rating) 10 or more Faults = 10 (rating)				Faulting is a difference of elevation across transverse joints or cracks. Severity of faulting is not measured directly, but is reflected in the overall riding quality. Extent of faulting is rated relative to number of faulted joints or cracks in 200 foot section. Cracks less than 18 inches in length are not considered.
Longitudinal Joint Separation	0-10	See Defect Description				Joint separation is the widening of the longitudinal joint between the PCC lanes. Extent and severity of joint separation are not directly rated since once a joint has opened to ¼ inch wide or wider it has generally been rated. If joint(s) unsealed or not tightly sealed a higher number should be assigned to the defect. (Both joints bounding the lane should be considered)
Patches	0-10	0% = 0 (rating) through 100% = 10 (rating)				Patches are temporary or permanent surface repairs to damaged pavement. Rate extent and condition of patching. Rate extent relative to the percentage of pavement surface area in 200 foot section patched. Rate condition of patching from good condition, performing satisfactorily to poor condition, significantly affecting ride quality. A higher rating should be assigned to poor condition patches.
Pavement Breakup	0-10	0% = 0 (rating) through 100% = 10 (rating)				Development of cracks in pavement which divide it into two or more pieces. Rate the extent and severity of breakup. Extent is rated relative to the percentage of surface area of 200 foot section that is broken up. Severity is rated relative to the stage of cracking from light pavement breakup which is initial stage cracking to severe pavement breakup which is corner cracking or fragmented pavement. A higher rating should be assigned to severe cracking.
Pumping	0-10	If no pumping is observed rate as 0. If any amount of pumping is observed rate as 5-10				Pumping is the ejection of water and fine materials under pressure through cracks under moving loads. If pumping exists anywhere in the 200 foot sample unit it is counted as occurring.
Overall Riding Quality	0-10	Roughness No. 2000 - 2250 = 0-3 (rating) Roughness No. 2251 - 2570 = 4-7 (rating) Roughness No. 2571 - 3800 = 8-10 (rating)				Roughness number appears on reverse side and is used to assign rating.

Sum of Defect Ratings _____

Condition Rating = 100 - Sum of Defect Rating

= 100 - _____

Condition Rating = [ ]

Comments _____

Table E1. Proposed new rating scales for Indiana.

Rigid Pavement Defects

25 - D-Cracking and Transverse Joint Condition
25 - All other types of cracking
25 - Patches and Slab breakup
15 - Pumping
.0 - Edge joint and shoulder condition

Flexible Pavement Defects

30 - All types of cracking
25 - Rutting and shoving
30 - Patches
10 - Pumping
5 - Edge joint and shoulder condition

APPENDIX F. Questionnaires for experts

Questionnaire No. 1

Introduction

The initial screening of road sections in determining needs for major maintenance is road roughness as measured by the roadmeters. Roughness data are calibrated (or correlated) with data from road rating panels in which a variety of road users cast their respective subjective judgment about the "quality" of the different pavement sections.

In the past, data from rating panels were lumped and no distinction was made between responses from different people. It has been suggested that raters with different backgrounds and experience will view pavement condition in different ways. This is consequent to the fact that urgency of maintenance, to some extent, depends on the likelihood that the pavement surface conditions suggest "serious" internal problems, that the conditions represent "serious" traffic hazards and whether the difficulties warrant correction in light of traffic volumes carried by the section. Each rater, then, has a different perspective towards the question of pavement "quality", and our objective is to gather highway experts' subjective opinions about raters' perceptiveness on pavement serviceability.

Thus, we seek your judgment. We have prepared a series of questions to allow us to assemble your judgment with those of others regarding the relative weight to be assigned to individual

ratings, when road sections' Present Serviceability Ratings (PSR) are being established.

1. Do you agree that judgments of some raters could deserve more weight when assembling panel data, depending on their perceptiveness of the serviceability question?

Yes No

2. If different weights are to be attached to the judgments, basically it is convenient to group them according to their backgrounds so that a single weight could be attached to each group. Table 1 describes the professional backgrounds and the ages of one such rating panel. One possible grouping for this panel is presented in Table 2. Do you agree with this grouping?

Yes No

If your response is 'No' please proceed to Q.7.

IF YOU AGREE WITH THE BASIC GROUPING.

3. If judgments of different raters deserve different relative weightings, on a scale of 00 to 2.0 what such levels would you assign to each group in determining Pavement Serviceability Ratings from this panel? (Assign a value such as 0.5 or 1.1 or 1.5 depending on the subjective perceptiveness you place on each group's judgment).

Group	Relative weight
A	
B	
C	

OR

Do you believe that each group's judgment should be viewed exactly the same in obtaining the PSR?

Yes No

4. After the basic grouping do you agree that categorization according to experience and other factors within a group is appropriate as shown in Table 2?

Yes No

5. If the answer to Question 4 is 'Yes', now we seek your subjective opinion on relative weighting factors to be assigned for the judgments of the respective sub-groups (taken as a unit) in a scale of 0.0-1.0?

Sub Group	Weighting Factor
Minor	
Major	

6. If the answer to Question 4 is 'No' then do you wish to see some changes between minor and major subgroups?

Yes No

If 'Yes' please indicate them on Table 2. (Indicate the rater to be shifted to the other sub-group by placing a '*' in front of him.) Further, what relative weighting factors would you assign for your modified sub-groups (each considered as a unit)

Sub-Group	Factor
Minor	
Major	

OR

Do you have other suggestions for categorization within a group?

Yes No

If 'Yes' please indicate them on the provided sheet.

Please proceed to Question No. 20.

IF YOU DISAGREE WITH THE BASIC GROUPING

7. Do you feel that a reassembly should be performed keeping the main groups the same?

Yes No

If the response is 'No' please proceed to Question No. 13.

8. If the answer to Question 7 is 'Yes' please indicate on Table 2 which rater you would shift and to where shifted. (Indicate this by writing the new group name in front of the rater.)

9. With your modified groups, please indicate your suggested relative weighting levels in the same manner as per instructions in Question 3. Use the table below to record your responses.

Group	Relative Weight
A	
B	
C	

10. Do you agree with the idea of categorizing within a group according to experience (age) if they are of the same professional types or according to their interest in pavements if they are of different background?

Yes No

11. If the answer to Question 10 is 'Yes' indicate the distribution of raters you would use for major and minor sub-groups for each of your groups. (Indicate this by writing M - for Major or m - for minor in front of the raters you have shifted in Table 2.)

12. For your sub-groups (each considered as a unit), on a scale of 0.0 - 1.0, what relative weighting levels would you attach to each?

Sub-group	Weighting Factor
Major	
Minor	

Please proceed to Question No. 20.

13. If the answer to Question 7 was 'No', are you of the opinion that more main groups should be formed according to their backgrounds?

Yes No

14. If the answer to Question 13 is 'Yes' please indicate main groups you would propose [Indicate the group name in front of each rater in Table 1.]

With your newly formed groups please answer Question 15, 16, 17 and 18 as per instructions in Questions 9, 10, 11 and 12, respectivel

15.

Group	Relative Weight
A	
B	
C	
D	
E	

16. Yes No

17. Use Table 1 for your responses.

18. Sub-group Weighting Factor

Major

Minor

*19. If the response to Question 1 was 'No' please indicate why not on the sheet provided.

FINALLY

20. A list of Highway users with various backgrounds who could be members of serviceability rating panels in general, is shown in Table 3 along with their ages. Please indicate in the space provided, on a scale of 0.0-2.0 the relative weights you would place in each raters judgment, in your subjective opinion.

1	Highway Research Engineer	(20-29)
2	Training Officer (Administrative)	(40-49)
3	Highway Research Engineer	(30-39)
4	"	(50-59)
5	"	(20-29)
6	Engineering Asst. Supervisor	(30-39)
7	Mechanic	(30-39)
8	Secretary	(30-39)
9	Highway Research Engineer	(50-59)
10	Highway Engineering Assistant	(20-29)
11	"	(20-29)
12	Electronics Technician	(40-49)
13	Bituminous Laboratory Technician	(20-29)
14	Highway Engineering Assistant	(30-39)
15	"	(20-29)
16	"	(50-59)

A Group		B Group		C Group	
Profession	Age	Profession	Age	Profession	Age
Research Eng.	50-59	Engineering Asst.	50-59	Lab. Technician	20-29
Research Eng.	50-59	Eng. Asst. Super.	30-39	Mechanic	30-39
				Electronics Technician	40-49
Research Eng.	30-39	Eng. Assistant	30-39	Training Off.	40-49
Research Eng.	20-29	Eng. Assistant	20-29	Secretary	30-39
Research Eng.	20-29	Eng. Assistant	20-29		
		Eng. Assistant	20-29		

	Under 40 yrs	Over 4
Highway Research Engineer		
Civil Engineers associated with highways		
Other Civil Engineers		
Mechanical Engineers		
Other Engineers		
Engineering Assistants associated with highways		
Other Civil Engineering Assistants		
Non-Civil Engineering Assistants		
Civil Engineering laboratory technicians		
Non-Civil Engineering Technicians		
Truck Driver		
Highway Research Administrator		
Other Professionals		
Working road users		
Housewives		

Questionnaire No. 2

Our previous questionnaire addressed the formation of a pavement serviceability rating (PSR), which incorporates the different degrees of perceptiveness of different raters.

Our next objectives are to correlate our PSR with roadmeter reading, obtain the PSI for pavement sections and to compare the PSI with the acceptable serviceability index (ASI) in rank-ordering pavements according to rideability.

In this questionnaire we seek your judgment on the inherent variability of the roadmeter and on the acceptable serviceability indices. These will allow us to develop a complete PSI model, while including all of the previous provisions for different degrees of perceptiveness and human uncertainty.

1. The Indiana Department of Highways, and also many other highway departments compare performances of pavements and set maintenance priorities based on the pavement serviceability index (PSI), as derived from the correlation of the roadmeter reading and the pavement serviceability rating (PSR). PSI takes values on a scale of 0.0-5.0. The PSI history of a pavement section shows that deterioration of the pavement quality occurs with time, leading to a sharp decrease in PSI below which the pavement condition becomes a traffic hazard. This hazardous stage is not sharply defined, and different people will recognize it at slightly different stages as their perceptions of the cost of

rehabilitation and the dangers involved with such a pavement differ.

a) Above what value of PSI, in your opinion, is a primary (major) pavement totally acceptable for traffic? (This is the Acceptable Serviceability Index or ASI.) Please answer the same question for a secondary pavement.

Type of Pavement	ASI
Primary	
Secondary	

b) Below what PSI value in your opinion is a primary pavement totally inadequate? (This is the Non-acceptable Serviceability Index or NASI.) Please answer the same question for a secondary pavement.

Type of Pavement	NASI
Primary	
Secondary	

2. Pavement Serviceability Rating is the combined judgment by the rating panel of the quality of the pavement. Each rater assigns a value for the section on the scale of 0.0-5.0 according to his or her perception of the serviceability of a pavement section. The rater's judgment is an imprecise number since if he or she is given more trials under similar conditions, repetitions of

the previous rating cannot be expected in general. In other words, since the judgment is purely subjective, he or she obviously gives considerable support to a domain around the assigned value. Therefore, it is believed that an individual opinion is better represented by such a range rather than a discrete number.

a) Do you agree with this idea?

b) If so, what in your opinion is the interval that such a rating can be within? [For example, if a rating of 2.5 can, in your opinion, be any value between 2.2 and 2.8, the required interval is 0.6.]

3. Inspection of roadmeter readings obtained from repeated trials on a given pavement section reveals a significant scatter. This may be mainly due to the inability of the roadmeter to replicate the same path every time it scans the contract section.

We can consider the roadmeter reading as supporting a range of values rather than a discrete value, in order to account for this imprecisely defined roughness.

Let us assume that the count obtained for a certain section is 500. If it is thought that a range of 10% of this count represents the variability of the reading, it may be appropriate to represent the roughness of that section by the interval 450-550, with the highest belief attached to 500.

a) Do you agree with this idea?

b) What such interval in your opinion is suitable to characterize a roadmeter reading? (Please provide the interval as a percentage of the reading.)

4. It is known that the gas tank level, driver characteristics and changes in vehicle speed have some influence on the roadmeter reading.

a) For a certain driver, assuming that the vehicle is driven at the standard speed, what is, in your opinion, the variability in roadmeter reading induced by the possible variations in the gas tank level. [Ex. For a roadmeter reading of 600, if the gas tank level changes vary the roadmeter reading within 594 and 606, then the required range is 2%.]

b) For a certain driver, assuming no changes in the gas tank level, in your opinion what range of variation is possible in the roadmeter reading due to any changes in vehicle speed from the standard speed of 50 mph? Please respond according to the example in 4.a.

c) Assuming the vehicle is driven at the standard speed and that there are no gas tank level changes, in your opinion what range of values is possible for a certain roadmeter reading due to possible variations in driver characteristics (i.e., unsteadyness, etc.)? Please respond according to the example in 4.a.

Questionnaire No. 3

Our previous questionnaires dealt with the determination of present serviceability rating (PSR) and acceptable serviceability index (ASI). These determinations include the use of the weights in data manipulations to account for different degrees of perceptiveness of different panel members rating a pavement. Uncertainty inherent in the roadmeter reading was also considered.

Now let us assume that pavement sections have been sorted out into those having acceptable or non-acceptable serviceability indices. For those having acceptable values, it is still possible that they require speedy maintenance to account for in-service behavior that it not included in the context of the PSR (rideability) concept.

For the pavements having acceptable serviceability indices, the next step in the evaluation system is categorization according to the level of skid-resistance. In the State of Indiana, skid-resistance of pavements is measured by the skid-tester and the friction number obtained is 100 x the coefficient of friction and can be any value between 0 and 100.

The friction number is found to be influenced by a number of factors such as temperature, rainfall and the speed of the tester. The changes due to these factors are so irregular that even the current statistical methods have not succeeded in identifying a systematic variation. In the presence of these variables, a

precise friction number cannot be defined. Due to this lack of precision, the skid-resistance of a contract section leads to system uncertainty.

The purpose of this questionnaire is to gather subjective information from experts, regarding the friction number variability and the acceptable friction numbers to ensure safety of traffic.

According to your judgment, above what friction number can the traffic move without significant risk of skidding? If you believe that this value depends on the pavement type, please indicate your values corresponding to each pavement type in the relevant spaces below.

Type of Pavement	FN
Asphalt	
Overlay	
CRCP	
JRCP	

According to your judgment, below what friction number is resurfacing necessary to prevent skidding? Please fill the following table as per instructions in the above question.

the previous rating cannot be expected in general. In other words, since the judgment is purely subjective, he or she obviously gives considerable support to a domain around the assigned value. Therefore, it is believed that an individual opinion is better represented by such a range rather than a discrete number.

a) Do you agree with this idea?

b) If so, what in your opinion is the interval that such a rating can be within? [For example, if a rating of 2.5 can, in your opinion, be any value between 2.2 and 2.8, the required interval is 0.6.]

3. Inspection of roadmeter readings obtained from repeated trials on a given pavement section reveals a significant scatter. This may be mainly due to the inability of the roadmeter to replicate the same path every time it scans the contract section.

We can consider the roadmeter reading as supporting a range of values rather than a discrete value, in order to account for this imprecisely defined roughness.

Let us assume that the count obtained for a certain section is 500. If it is thought that a range of 10% of this count represents the variability of the reading, it may be appropriate to represent the roughness of that section by the interval 450-550, with the highest belief attached to 500.

a) Do you agree with this idea?

b) What such interval in your opinion is suitable to characterize a roadmeter reading? (Please provide the interval as a percentage of the reading.)

4. It is known that the gas tank level, driver characteristics and changes in vehicle speed have some influence on the roadmeter reading.

a) For a certain driver, assuming that the vehicle is driven at the standard speed, what is, in your opinion, the variability in roadmeter reading induced by the possible variations in the gas tank level. [Ex. For a roadmeter reading of 600, if the gas tank level changes vary the roadmeter reading within 594 and 606, then the required range is 2%.]

b) For a certain driver, assuming no changes in the gas tank level, in your opinion what range of variation is possible in the roadmeter reading due to any changes in vehicle speed from the standard speed of 50 mph? Please respond according to the example in 4.a.

c) Assuming the vehicle is driven at the standard speed and that there are no gas tank level changes, in your opinion what range of values is possible for a certain roadmeter reading due to possible variations in driver characteristics (i.e., unsteadyness, etc.)? Please respond according to the example in 4.a.

Questionnaire No. 3

Our previous questionnaires dealt with the determination of present serviceability rating (PSR) and acceptable serviceability index (ASI). These determinations include the use of the weights in data manipulations to account for different degrees of perceptiveness of different panel members rating a pavement. Uncertainty inherent in the roadmeter reading was also considered.

Now let us assume that pavement sections have been sorted out into those having acceptable or non-acceptable serviceability indices. For those having acceptable values, it is still possible that they require speedy maintenance to account for in-service behavior that it not included in the context of the PSR (rideability) concept.

For the pavements having acceptable serviceability indices, the next step in the evaluation system is categorization according to the level of skid-resistance. In the State of Indiana, skid-resistance of pavements is measured by the skid-tester and the friction number obtained is 100 x the coefficient of friction and can be any value between 0 and 100.

The friction number is found to be influenced by a number of factors such as temperature, rainfall and the speed of the tester. The changes due to these factors are so irregular that even the current statistical methods have not succeeded in identifying a systematic variation. In the presence of these variables, a

precise friction number cannot be defined. Due to this lack of precision, the skid-resistance of a contract section leads to system uncertainty.

The purpose of this questionnaire is to gather subjective information from experts, regarding the friction number variability and the acceptable friction numbers to ensure safety of traffic.

According to your judgment, above what friction number can the traffic move without significant risk of skidding? If you believe that this value depends on the pavement type, please indicate your values corresponding to each pavement type in the relevant spaces below.

Type of Pavement	FN
Asphalt	
Overlay	
CRCP	
JRCP	

According to your judgment, below what friction number is resurfacing necessary to prevent skidding? Please fill the following table as per instructions in the above question.

Type of Pavement	FN
Asphalt	
Overlay	
CRCP	
JRCP	

2. Research on the skid-tester has revealed that the friction number of a pavement section is affected by the climatic conditions (temperature and rainfall differences) and, the speed of the skid-tester, which are independent of each other.

The friction number of a pavement section is obtained under given climatic conditions, defined by temperature and rainfall. If this number is to represent the skid characteristics of that section under any climatic condition (assuming that there are no vehicle speed changes), it may be more appropriate to indicate an interval instead of a unique number.

a) Do you agree with this idea?

If your response is 'No' please proceed to 2(ii).

b) What such friction number interval should be specified for a certain friction number according to your judgment?

You may use the following table to indicate your answer, as a percentage of the friction number. [For example, if you believe that for a pavement indicating a friction number of 50,

the friction numbers due to possible climatic changes could be within 45 and 55, the required interval is 20%.]

Pavement Type	Friction Number
Asphalt	
Overlay	
CRCP	
JRCP	

ii) Suppose that the friction number of a pavement section is obtained for a given vehicle speed, which might be different from the standard vehicle speed. If that number is to account for any such speed changes from the standard vehicle speed (assuming that there are no climatic changes) it may be more appropriate to specify a friction number interval along with the above friction number.

a) Do you agree with this idea?

b) What such interval, in your opinion, should be specified for a certain friction number?

You may use the following table to indicate your answer, as per instructions in 2(i).

Pavement Type	Friction Number
Asphalt	
Overlay	
CRCP	
JRCP	

Questionnaire No. 4

Three questionnaires have been previously sent out by us with the objective of collecting highway engineers' expertise on pavement evaluation. Those questionnaires addressed the incorporation of the raters' perceptiveness in serviceability rating, acceptable serviceability indices, roadmeter variability and the variability associated with the friction number. Techniques are available to include this expert knowledge, once gathered, in the pavement evaluation system using fuzzy sets mathematics.

In assigning priorities for maintenance, pavement sections are first separated into two categories, with acceptable and unacceptable PSI values. The sections with acceptable roughness are then tested for skid-resistance and by means of an acceptable friction number they are further sub-divided into two categories: acceptable and unacceptable skid levels. Currently, the IDOH is planning to develop a procedure to test the sections with unacceptable roughness (PSI) for deflections in order to obtain overlay design thicknesses.

Indiana Department of Highways uses the dynaflect for deflection testing. Variability is also associated with the dynaflect reading which is vulnerable to changes in climatic conditions. Furthermore, the inability to repeat a reading at a particular test position also contributes to this.

In this questionnaire we address the problem of variability associated with the deflection reading. We seek your judgment on

the possible extent of these variations to enable us to incorporate them in the evaluation system.

1. Results of dynaflect tests at the same location on a given section reveals a wide scatter. Thus, it may be appropriate to characterize the deflection by a range of values. Assuming no climatic changes, in your opinion what range of variation would you expect in the dynaflect reading at the same location on repeated trials? [Ex.: If the dynaflect reading at a certain location is 8 mils and you believe that in repeated measurements it may lie between 7.6 and 8.4 mils, then the required range is 10%.]

Please indicate your answer as a percentage in front of each pavement type given below.

Pavement Type	Possible range of variation (%)
Asphalt	
Overlay	
CRCP	
JRCP	

2. Deflections measured at the edge of a pavement are considered more critical because deterioration usually initiates at the edge. It is, however, more convenient to measure the center deflections, which are later correlated to the edge deflections. What factors would you propose to be used in converting center

deflections to those at the edge? Please indicate your answer corresponding to each pavement type.

Pavement type	Edge def./center def.
Asphalt	
Overlay	
CRCP	
JRCP	

Currently these factors are obtained by statistical methods. But we notice that the analysts' subjective judgment invariably is involved in such procedures. For example, when using the analysis of variance (ANOVA) method to get the conversion factors, only the "80th percentile" values of the variances of each set of deflections are considered. Imprecision is also introduced due to the difficulty in identifying the "edge", especially for deteriorated pavements, since the edge deflection is defined as that at a distance of 2' from the edge. Therefore, it is proposed that these factors be replaced by appropriate tolerance intervals in order to handle the above mentioned human and system uncertainties involved in the edge-center deflection correlation.

Do you agree with this idea?

If so, what tolerances should be attached to the above factors? Please indicate your answer as a percentage of the factor, in the following table. [Ex: If you think it is appropriate to use a factor of 2.0 for the ratio of edge deflection/center

deflection for flexible pavements, and when the above mentioned uncertainties are taken into account, if, in your opinion, this factor could be any value between 1.5 and 2.5 the required tolerance for flexible pavements is ± 25%.]

Pavement Type	Tolerance (%)
Asphalt	
Overlay	
CRCP	
JRCP	

3. In Indiana, deflections measured during the spring thaw period are found to be the most critical for overlay design purposes. But in practice it is difficult to scan all the deteriorated pavement sections with the dynaflect within this short period. Thus, deflections are usually measured during Fall and later converted to the corresponding spring deflections. What factors would you suggest using for this conversion? Please indicate your answer corresponding to each pavement type.

Pavement Type	Spring def./center def
Asphalt	
Overlay	
CRCP	
JRCP	

These factors are presently obtained by statistical correlation procedures. But it is seen that for some pavement types the correlation coefficients are relatively low (i.e., in the order of 0.2, etc.). This scatter of data imparts imprecision on the factors so derived. Therefore, it seems more appropriate to replace these factors by suitable tolerance intervals.

Do you agree with this idea?

If so, in your opinion what tolerances should be attached to the above factors to make them more representative of the possible values. Please indicate your answer as a % of the factor in the following table. [Ex: If it is appropriate to use a factor of 1.5 for the ratio of spring deflection/fall deflection for asphalt pavements, and when the above mentioned imprecision inherent in deriving these factors is taken into account, if, according to your judgment, this factor could be any value between 1.35 and 1.65 the required tolerance for asphalt pavements is ± 10%.]

Pavement Type	Tolerance (%)
Asphalt	
Overlay	
CRCP	
JRCP	

Questionnaire No. 5

We have previously sent four questionnaires which dealt with various stages of our research on application of fuzzy sets mathematics in pavement evaluation. They addressed such areas as subjectivity involved in pavement serviceability rating, acceptable serviceability and variability associated with roadmeter, skid-tester and dynaflect.

Once pavements are screened for serviceability (roughness) at the network level, skid-testing is scheduled for those pavements with acceptable roughness. On the other hand, unacceptably rough pavements are subjected to deflection testing for structural adequacy. The next stage is to investigate the distress condition of the pavements which fall into the latter category.

Indiana Department of Highways is currently setting out a procedure to be adopted in distress surveys. A crew is to carefully examine and roughly estimate the extent and severity of different pavement defects in a selected length of a section at a designated mile post. Separate instruction sheets have been prepared for flexible and rigid pavement types to aid the crew in assigning ratings. The combined rating is known as the pavement condition rating (PCR). However, human based uncertainty enters the PCR by way of imprecision of measurements and subjective judgments. The purpose of this questionnaire is to seek your expertise in judgment type responses, on the magnitude of the

above mentioned uncertainties for various kinds of distresses, and the acceptable level of distress for different types of pavements.

1. Pavements which satisfy roughness and skid criteria will be ranked in the future rehabilitation priority list. Since these pavements also can exhibit some distress, it will be worthwhile to conduct distress surveys on them too. This certainly will provide an additional criterion for their ranking.

Do you agree with this idea?

2. Pavement distress inspection crew will rate the sections for different defects, on different scales. These ratings are finally added up and subtracted from the maximum rating of 100, to obtain the pavement condition rating (PCR). Thus, a PCR for any pavement can be any value between 0 and 100, with 100 being that for a "defectless" pavement.

If we were to categorize pavements into two classes: those with acceptable distress and others with unacceptable distress, a terminal (or acceptable) PCR has to be defined. In your opinion what such acceptable PCR is suitable for this classification? Please indicate this value for both the pavement types and facilities, in the following table.

facility type	Primary	Secondary
flexible		
rigid		

3. The extent of certain types of defects are determined by the percentage of area in a 200 foot section, denoting the defect. Alligator cracks, block cracking, shoving and patches are such defects for flexible pavements while D-cracking, patching and pavement break-up correspond to rough pavements. Since the area determination is not precise, a 'margin of error' is introduced in the rating. For example, when a crew rates alligator cracking its support may spread over a range between 4.5 and 5.5 due to the uncertainty in the area determination, although it may assign a value of 5.0. Thus, the margin of error is ± 0.5.

Do you agree with this idea?

If so, according to your judgment, what such margins would you assign for the following defect types?

	Type	Scale	Margin of Error (±)
Flexible	alligator cracks	0-10	
	block cracking	0-5	
	shoving	0-10	
	patching	0-10	
Rigid	D-cracking	0-10	
	patching	0-10	
	pavement break-up	0.10	

4. According to the rating instructions, most defects are to be rated in proportion to their severity. All forms of cracking, patching in all types of pavements and pavement slab break-up in rigid pavements fall into this category. Although in the case of cracks, the severity is determined by the crack width, assignment of a 'higher' rating for 'severe' conditions is purely subjective. Due to this reason, attaching a suitable interval for a rating may be appropriate to look after the vagueness introduced by severity.

As an example, assume that crew members rate a section as 2.3 for transverse cracking considering the extent only. Then, once they observe that the crack is severe (width being greater than 1/4") they are compelled to go for a higher rating. They might consider a range between 3.0 and 3.5 before deciding on

3.25, to which they give the highest support. In this case the above mentioned interval is defined by ± 0.25.

Do you agree with this idea?

If so, in your opinion, what such intervals should be attached for the following defects?

	Type	Scale	Interval (± x)
Flexible	transverse cracks	0-5	
	longitudinal cracks	0-5	
	patching	1-10	
Rigid	pavement break-up	0-10	
	patching	0-10	
	transverse cracks	0-10	
	longitudinal cracks	1-10	
	D-cracking	0-10	

Questionnaire No. 6 (i)

This is the sixth one in a series of questionnaires being forwarded to highway engineers to obtain their expertise on a number of subjective areas in pavement management. Our previous questionnaires addressed areas such as pavement serviceability rating, acceptable serviceability and variability associated with roadmeter, skid-tester and dynaflect.

At present, in the state of Indiana four kinds of performance data are collected at the highway network level. At first, all the pavements are scanned with the roadmeter. Then skid tests are conducted for pavements with acceptable roughness, while dynaflect and distress surveys are carried out on the others. Using these measurements, decisions have to be made concerning priorities for maintenance or rehabilitation.

In Indiana, as in many other states, three primary categories of pavements are identified. The first category has those pavements which present traffic hazards due to inadequate skid resistance, while pavements with unacceptable roughness constitutes the second category. The third and the final category contains the ones with acceptable skid-resistance and roughness.

Immediate attention is given to the first category whereas pavements in the second category are prepared for maintenance during the current year considering a fixed budget. Pavement sections that belong to the third category are prioritized for future rehabilitation, using remaining service life. As it is

practiced in most states (including Indiana to a certain extent), current priority lists (for categories 1 and 2) are prepared using applicable performace parameters (roughness, structural adequacy, skid-resistance and distress data) and traffic data. Since the basis for prioritization for category 3 is different, it will not be discussed here, any further.

It is understood that decisions regarding relative priority levels becomes subjective and complex if all the relevant parameters are considered. In order to avoid this, highway experts can be made to respond to less complicated questions. Then these responses can be methodically combined using fuzzy decision techniques in arriving at a rank for each pavement section. Thus, in this questionnaire we seek subjective responses from you for simple questions regarding priority levels for the first category.

(1) We know that friction and traffic data play a prime role in determining maintenance priorities for the above described first category of pavements. In your opinion are these two types of data adequate for this purpose?

If the answer is 'No', please indicate any other type of data which you think is required.

(2) If we assume that priorities can be numerically represented by values on a scale of 1 - 10 which is adopted by the Indiana Department of Highways, each combination of <FN, ADT> will determine a value on that scale. This value results from

subjective judgment of highway maintenance decision makers.

For example if we assign a range of values of 9.1 - 9.5 as the priority for the combination <FN = 20, ADT = 8000>, then the priority range for the case of <FN = 30, ADT = 6000> could be about 8.0 - 8.5, which is less than the above.

In the table below, as few such combinations of attribute values are provided. In the appropriate places please indicate priority value ranges (in a scale of 1 - 10) which you think will best describe the maintenance urgency of a pavement that has such attribute values.

	FN			
ADT	10	20	30	40
1000				
6000				
10000				

Questionnaire No. 6 (ii)

This is the sixth one in a series of questionnaires being forwarded to highway enginers to obtain their expertise on a number of subjective areas in pavement management. Our previous questionnaires addressed areas such as pavement serviceability rating, acceptable serviceability and variability associated with roadmeter, skid-tester and dynaflect.

At present, in the state of Indiana four kinds of performance data are collected at the highway network level. At first, all the pavements are scanned with the roadmeter. Then skid tests are conducted for pavements with acceptable roughness, while dynaflect and distress surveys are carried out on the others. Using these measurements, decisions have to be made concerning priorities for maintenance or rehabilitation.

In Indiana, as in many other states, three primary categories of pavements are identified. The first category has those pavements which present traffic hazards due to inadequate skid resistance, while pavements with unacceptable roughness constitutes the second category. The third and the final category contains the ones with acceptable skid-resistance and roughness.

Immediate attention is given to the first category whereas pavements in the second category are prepared for maintenance during the current year considering a fixed budget. Pavement sections that belong to the third category are prioritized for future rehabilitation, using remaining service life. As it is

practiced in most states (including Indiana to a certain extent), current priority lists (for categories 1 and 2) are prepared using applicable performace parameters (roughness, structural adequacy, skid-resistance and distress data) and traffic data. Since the basis for prioritization for category 3 is different, it will not be discussed here, any further.

It is understood that decisions regarding relative priority levels becomes subjective and complex if all the relevant parameters are considered. In order to avoid this, highway experts can be made to respond to less complicated questions. Then these responses can be methodically combined using fuzzy decision techniques in arriving at a rank for each pavement section. Thus, in this questionnaire we seek subjective responses from you for simple questions regarding priority levels for the second category.

(1) Once pavements have been categorized according to their PSI (roughness) and friction, we propose that the pavements in second category (with unacceptable PSI) be prioritized using the results of their distress, dynaflect and traffic surveys. In other words, prioritization of second category of pavements can be done with values of PCR, deflections, and ADT, available.

Do you agree with this idea?

If your answer is 'yes' please proceed to question No. 2.

If your answer is 'no', what other parameters are necessary in your opinion for the prioritization of the second category of pavements ?

(2) If we assume that priorities can be numerically represented by values on the scale of 1 - 10, as adopted by the IDOH presently, each combination of <PCR, ADT, deflection> will determine a value on that scale. This value results from subjective judgment of highway maintenance decision makers.

For example, if we assign a range of values of 9.0 - 9.5 for the combination <PCR = 10.0, ADT = 8000 and def. = 1.1 mils>, then a possible priority value range for the combination <PCR = 25.0, ADT = 6000 and def. = 1.1 mils> could be 7.5 - 8.0.

In the tables below, we provide a few such combinations of attribute values. In the appropriate places please indicate a priority value range (in a scale of 1 - 10) which you think will best describe the maintenance urgency of a pavement that has such attribute values.

ADT = 3000

PCR def. (mils)	10	25	50	70
1.0 2.0				

ADT = 6000

PCR def. (mils)	15	30	65	75
1.25 2.25				

ADT = 10000

PCR def. mils)	20	35	45	60
1.5 2.5				

Questionnaire No. 7

This is the seventh in a series of questionnaires sent to highway experts to seek their opinion on certain areas of pavement management that involves human based uncertainty. Four kinds of data are collected to determine performance of highway pavements in Indiana, namely roughness, skid-resistance, distress and deflection data. Decisions concerning priorities for maintenance are made based on these data as well as subjective judgment of highway engineers.

In Indiana, as in many other states, three categories of pavements are identified for maintenance at different stages. The first category has those pavements which present traffic hazards due to inadequate skid resistance, while pavements with unacceptable roughness constitutes the second category. All the pavement sections falling under the above categories need immediate attention and are dealt with accordingly. The two preceding questionnaires addressed subjectiveness associated with prioritization of this kind of pavements.

The third and the final category contains the ones with acceptable skid-resistance as well as roughness. In this questionnaire we are concerned with the subjectivity involved in the prioritization of these, still serviceable pavements, for future rehabilitation.

(1) The Pavement Management Task Force of the Indiana Department of Highways has proposed collecting condition data on

all pavement sections; instead of only on unacceptably rough sections, as one of its short term objectives. This will provide yet another criterion for prioritization of the third category of pavements.

Do you agree with this idea?

(2) Many variables have to be considered in ranking pavement sections for future maintenance. The most important ones are remaining PSI and friction lives and ADT of the section. Since ADT is the prime factor governing both types of service lives, once we consider the service lives as two pertinent attributes, inclusion of ADT seems redundant. Therefore, we suggest that prioritization of pavements in Category 3 can be done using only the service lives as attributes.

Do you agree with this idea?

(3) We know that prioritization of highway pavements for future rehabilitation involves multi-attribute decisions, i.e., assigned rankings depend on the remaining PSI life and remaining friction life of the particular pavement. Thus, if we assume that priorities can be represented numericaly by the scale 1 - 10, each 2-tuple <PSI life, FN life> will determine a value on that scale. With the knowledge of the above two parameters for the section, a decision maker would be able to assign a ranking to the section, using his experience.

As an example, if we assign a priority range of values of 8.0 - 8.5 for the combination

<PSI life = 3.2 yrs, FN life = 2.2 yrs>,

then the combination of

<PSI life = 2.2 yrs, FN life = 3.2 yrs>,

would have to be assigned a lower priority value, say 6.8 - 7.0.

In the table below, we provide a few such combinations of attribute values. In the appropriate places, please indicate a priority value range (in a scale of 1 - 10), which you think will best describe the future maintenance urgency of a pavement that has such attribute values.

FN LIFE \ PSI LIFE	2.0	4.0	6.0	8.0
1.0				
3.0				
5.0				
7.0				

APPENDIX G. Results of numerical example

Table H2. Fuzzy PSI and acceptability indices

pavement section no: 1

PSI	membership
2.100	0.015
2.200	0.318
2.300	0.318
2.400	0.318
2.500	0.768
2.600	1.000
2.700	0.672
2.800	0.414
2.900	0.019
3.000	0.019

index .300

pavement section no: 2

PSI	membership
2.100	0.094
2.200	0.375
2.300	0.769
2.400	0.769
2.500	0.768
2.600	1.000
2.700	0.564
2.800	0.253

index .231

pavement section no: 3

PSI	membership
2.100	0.411
2.200	0.411
2.300	0.934
2.400	1.000
2.500	0.758
2.600	0.730
2.700	0.557
2.800	0.045

index .066

Table H2. Fuzzy PSI and unacceptability indices

pavement section no: 1

PSI	membership
2.100	0.015
2.200	0.318
2.300	0.318
2.400	0.318
2.500	0.768
2.600	1.000
2.700	0.672
2.800	0.414
2.900	0.019
3.000	0.019

index .328

pavement section no: 2

PSI	membership
2.100	0.094
2.200	0.375
2.300	0.769
2.400	0.769
2.500	0.768
2.600	1.000
2.700	0.564
2.800	0.253

index .436

pavement section no: 3

PSI	membership
2.100	0.411
2.200	0.411
2.300	0.934
2.400	1.000
2.500	0.758
2.600	0.730
2.700	0.557
2.800	0.045

index .443

Table H3. Fuzzified deflections

pavement section no. 1

defltn.(10-4 in.)	membership
11.000	0.100
10.000	0.845
9.000	0.845
8.000	0.100

pavement section no. 2

defltn.(10-4 in.)	membership
34.000	0.
33.000	0.100
32.000	0.100
31.000	0.431
30.000	0.845
29.000	0.845
28.000	1.000
27.000	0.845
26.000	0.845
25.000	0.431
24.000	0.100
23.000	0.100

pavement section no. 3

defltn.(10-4 in.)	membership
22.000	0.100
21.000	0.431
20.000	0.845
19.000	1.000
18.000	0.845
17.000	0.431
16.000	0.100
15.000	0.

Table H4. Fuzzy PCR values

pavement section no. 1

PCR	membership
43.00	0.010
44.00	0.022
45.00	0.062
46.00	0.099
47.00	0.151
48.00	0.239
49.00	0.321
50.00	0.551
51.00	0.729
52.00	0.880
53.00	0.901
54.00	0.901
55.00	0.901
56.00	0.901
57.00	0.880
58.00	0.729
59.00	0.321
60.00	0.321
61.00	0.239
62.00	0.151
63.00	0.099
64.00	0.062
65.00	0.022
66.00	0.010

pavement section no. 2

PCR	membership
38.00	0.010
39.00	0.049
40.00	0.130
41.00	0.200
42.00	0.257
43.00	0.366
44.00	0.431
45.00	0.605
46.00	0.628
47.00	0.787
48.00	0.901
49.00	0.975
50.00	1.000
51.00	0.975
52.00	0.901
53.00	0.787

Table H4. (Continued)

54.00	0.628
55.00	0.605
56.00	0.431
57.00	0.366
58.00	0.257
59.00	0.200
60.00	0.130
61.00	0.049
62.00	0.010

pavement section no. 3

PCR	membership
20.00	0.006
21.00	0.029
22.00	0.062
23.00	0.130
24.00	0.228
25.00	0.321
26.00	0.470
27.00	0.545
28.00	0.545
29.00	0.605
30.00	0.628
31.00	0.729
32.00	0.837
33.00	0.879
34.00	0.879
35.00	0.879
36.00	0.879
37.00	0.879
38.00	0.879
39.00	0.879
40.00	0.837
41.00	0.729
42.00	0.628
43.00	0.605
44.00	0.545
45.00	0.545
46.00	0.470
47.00	0.321
48.00	0.228
49.00	0.130
50.00	0.062
51.00	0.029
52.00	0.006

Table H5. Relative priorities

section	rel. priority
1	0.850
2	0.500
3	0.160

APPENDIX H. Computer programs

```
c  PROGRAM ROAD
c  This program can be used to ;
c         i. form a technical PSR out of panel ratings
c        ii. fuzzify a roadmeter reading
c       iii. formulate a fuzzy relationship - PSR Vs.Roadmeter
c        iv. compose an equivalent PSR out of a roadmeter reading
       implicit real(a-z)
c
       dimension must(50),mx(50),mu(55),mus(120),accp(50)
       dimension mupr(50,120),z(120),ck(205)
c
       integer i,j,k,l,m,n,nn,b1,b2,11,np,nacc,nnn
c
c  Formation of a fuzzy technical PSR out of panel ratings
c
       do 15 i=1,50
       do 10 j=1,120
       mupr(i,j)=0.0
       accp(i)=0.0
10     continue
15     continue
c
       read *,ns,b
       do 2500 11=1,ns
c
       do 16 1=1,50
       must(1)=0.0
       mx(1)=0.0
16     continue
       do 18 1=1,55
       mu(1)=0.0
18     continue
c
       read *, ngrp
c
       do 300 nn=1,ngrp
c
       read *, nsub,alpha
c
          do 200 n=1,nsub
          read *, nurat,w
c
             do 100 M=1,nurat
             read *, p
             b1=10*b
c
                 do 50 i=1,b1
                 x=p-(i-1)/10.0
                 b2=0.5*b1
                 if ((i-1). gt. b2) go to 20
                 mx(i)=1-2*(((i-1)/10.0/b)**2)
                 go to 30
20               mx(i)=2*((((i-1)/10.0-b)/b)**2)
```

```
30                continue
c
                  j=10.0*x
                  k=10.0*(x+(l-1)/5.0)
c
                  mx(i)=w*mx(i)
                  if (j. lt. 0.0) go to 40
                  if (mx(i) .lt. mu(j)) go to 40
                  mu(j)=mx(i)
c
40                if (mx(i). lt. mu(k)) go to 50
                  mu(k)=mx(i)
50                continue
c
100            continue
200         continue
c
            do 280 l=1,50
            mu(l)=mu(l)**alpha
            if (nn. gt. l)go to 260
            must(l)=mu(l)
            go to 270
260         must(l)=must(l)*mu(l)
270          mu(l)=0.0
280         continue
c
300      continue
c
c  normalize psr for combination with rr
c
         maxmust=0.0
         do 400 l=1,50
         if (maxmust. gt. must(l)) go to 400
         maxmust=must(l)
400      continue
c
         if (maxmust. eq. 0.0) go to 500
         do 450 l=1,50
         must(l)=must(l)/maxmust
450      continue
c
500      continue
c
c  Fuzzification of the roadmeter reading
c
c
         do 600 l=1,120
         mus(l)=0.0
600      continue
c
         call meter (mus,z,ll,ck,rint)
c
c
```

```
c  Formulation of the PSR-Roadmeter relationship
c
c
       do 2400 i=1,50
          do 2300 j=1,120
          if (must(i). gt. mus(j)) go to 2200
          if (mupr(i,j). gt. must(i)) go to 2250
          mupr(i,j)=must(i)
          go to 2250
2200      if (mupr(i,j). gt. mus(j)) go to 2250
          mupr(i,j)=mus(j)
2250      ii=i/10.0
2300      continue
c
2400   continue
c
2500   continue
c
c
c  Input of acceptable PSI range
c
       read *,nacc
c
       do 2700 nnn=1,nacc
       read *, i,accp(i)
2700   continue
c
c
c  Composition
c
       read *,np
       do 7000 m=1,np
       write(6,6100) m
c
       call meter(mus,z,ll,ck,rint)
c
       do 3600 n=1,50
       must(n)=0.0
3600   continue
       do 4450 I=1,50
           do 4400 J=1,120
           if (mus(j). gt. mupr(i,j)) go to 4200
           if (must(i). gt. mus(j)) go to 4300
c
           must(i)=mus(j)
           go to 4300
c
4200       if (must(i). gt. mupr(i,j)) go to 4300
           must(i)=mupr(i,j)
4300       continue
4400       continue
c
4450   continue
```

```
c  normalize psi for comparison
c
       maxmust=0.0
       do 4800 i=1,50
       if (maxmust. gt. must(i)) go to 4800
       maxmust=must(i)
4800   continue
c
c
       write(6,6300)
c
       do 4900 i=1,50
       ii=i/10.0
       if (maxmust. eq. 0.0) go to 4810
       must(i)=must(i)/maxmust
4810   if (must(i). lt. 0.001) go to 4900
       write(6,4850) ii,must(i)
4850   format(f6.3,3x,f6.3)
4900   continue
5000   continue
c
c
c  Calculation of the comparison index
c
       truth=1.0
c
       do 6000 j=1,50
       must(j)=1.0-must(j)
       if (must(j). gt. accp(j)) go to 5500
       must(j)=accp(j)
5500   continue
c
       if (truth. lt. must(j)) go to 6000
       truth=must(j)
6000   continue
c
       write(6,6200)truth
6100   format(////,'pavement section no: ',i3,//)
6200   format(//,2x,'index',8x,f4.3)
6300   format(2x,'PSI',5x,'membership',/)
c
7000   continue
8000   continue
c
       stop
       end
c
c
c
       subroutine meter(mu,z,ll,ck,rint)
c
c
```

```
c  This subroutine is used to fuzzify the roadmeter reading for
c       i. imprecision of the measuring system.
c      ii. variations in the gas tank level.
c     iii. variations in the vehicle speed.
c      iv. variations in driver characteristics.
c
c
c
       implicit real(a-z)
       dimension k1(50),k2(200),ck(205),z(120),y(205),mu(120)
       integer i,j,k,m,n,ii,jj,kk,il,kkl,kk2,itr,ll
c
c  Read the input data : i. percent ranges of variation
c                        ii. number of intervals for discretization
c
       if (ll. gt. 1) go to 2200
c
       itr=1
       read *,r1,r2,n
       if (itr. eq. 1)go to 20
10     read *,r1
       itr=itr+1
        go to 30
20     an=n-1
       rint=r1/an
c
30     continue
c
c  Initialization
c
       do 200 j=1,200
       k2(j)=0.0
200    continue
       do 100 i=1,n
       k1(i)=0.0
100    continue
c
c  Assigning of membership values to the kernels.
c
       do 500 i=1,50
       rr1=(i-1)*rint
c
       if (r1. ne. 0.0) go to 250
       k1(i)=1.0
       go to 600
250    if (rr1. gt. r1) go to 600
       if (rr1. gt. 0.5*r1)go to 300
       k1(i)=1-2*((rr1/r1)**2)
       go to 500
300    k1(i)=2*(((rr1-r1)/r1)**2)
500    continue
c
600    n=i
```

```
      if (itr. gt. 1)go to 1100
c
      do 1000 j=1,50
      rr2=(j-1)*rint
c
      if (r2. ne. 0.0) go to 750
      k2(j)=1.0
      go to 1200
750   if (rr2. gt. r2)go to 1200
      if (rr2. gt. 0.5*r2)go to 800
      k2(j)=1-2*((rr2/r2)**2)
      go to 1000
800   k2(j)=2*(((rr2-r2)/r2)**2)
1000  continue
      go to 1200
c
1100  continue
c
      do 1150 j=1,200
      if (ck(j). lt. 0.001)go to 1200
      k2(j)=ck(j)
1150  continue
c
c
1200  m=j
c
      do 1250 k=1,200
      ck(k)=0.0
      y(k)=0.0
1250   continue
c
      do 2000 i=1,n
      ii=i-1
      do 1800 j=1,m
      jj=j-1
      kk=ii+jj
      k=kk+1
      k12=k1(i)*k2(j)
      if (ck(k). gt. k12) go to 1300
      ck(k)=k12
1300  continue
c
      if (ii. gt. jj)go to 1400
      kk=jj-ii
      go to 1500
1400  kk=ii-jj
1500  continue
      if (ck(k). gt. k12)go to 1800
      ck(k)=k12
1800  continue
2000  continue
c
```

```
      if (itr. ne. 3)go to 10
c
2200  continue
c
c  Read input roadmeter readings
c
      read *,x
c
c  find memberships of L and R halfs
c
c
      do 2500 k=1,200
      if (ck(k). le. 0.0)go to 2600
      kk1=2*k-1
      kk2=2*k
      y(kk1)=x+rint*(k-1)*x
      y(kk2)=x-rint*(k-1)*x
2500  continue
c
2600  continue
      kmax=k
      ymax=x+rint*(kmax-1)*x
      ymin=x-rint*(kmax-1)*x
c
c  select standard scale of  25 - 3000
c
      do 2800 il=1,120
      z(il)=0.0
      mu(il)=0.0
2800   continue
c
      do 4000 il=1,120
      z(il)=3025-il*25.0
      if (z(il). gt. ymax) go to 4000
      if (z(il). lt. ymin) go to 4100
      do 3600 k=1,kmax
      kk1=2*k-1
      kk2=2*k
      test=rint*x*0.5
      if (abs(z(il)-y(kk1)). gt. test)go to 3000
      mu(il)=ck(k)
      go to 4000
3000  if (abs(z(il)-y(kk2)). gt. test)go to 3600
      mu(il)=ck(k)
      go to 4000
3600  continue
4000  continue
4100  continue
c
      return
      end
```

```
c  PROGRAM SKID
c  This program is used to :
c       i. fuzzify the friction number for variations in temperature
c          and rainfall.
c      ii. fuzzify the friction number for vehicle speed variations.
c
c
c
       implicit real(a-z)
       dimension k1(50),k2(50),ck(205),z(105),y(205),mu(105)
       integer i,j,k,l,m,n,ii,jj,kk,nn,il,kk1,kk2
c
c  Read the input data : i. number of data sets
c                        ii. percent ranges of variation
c                       iii. number of intervals for discretization
c
c           (Note: if one variation is zero use the other
c                  as r1)
c
       read *,nn
       read *,r1,r2,n
       an=n-1
       rint=r1/an
c
c  Initialization
c
       do 100 i=1,n
       k1(i)=0.0
100    continue
c
       do 200 k=1,200
       ck(k)=0.0
       y(k)=0.0
200    continue
c
       do 250 il=1,100
       z(il)=0.0
       mu(il)=0.0
250    continue
c
c  Assigning of membership values to the kernels.
c
       do 500 i=1,n
       rr1=(i-1)*rint
c
       if (r1. ne. 0.0) go to 280
       k1(i)=1.0
       go to 600
280    if (rr1. gt. 0.5*r1)go to 300
       k1(i)=1-2*((rr1/r1)**2)
       go to 500
300    k1(i)=2*(((rr1-r1)/r1)**2)
500    continue
```

```
600     continue
c
        do 1000 j=1,50
        rr2=(j-1)*rint
c
        if (r2. ne. 0.0) go to 750
        k2(j)=1.0
        go to 1100
750     if (rr2. gt. r2)go to 1100
        if (rr2. gt. 0.5*r2)go to 800
        k2(j)=1-2*((rr2/r2)**2)
        go to 1000
800     k2(j)=2*(((rr2-r2)/r2)**2)
1000    continue
c
1100    m=j
        do 2000 i=1,n
        ii=i-1
        do 1800 j=1,m
        jj=j-1
        kk=ii+jj
        k=kk+1
        k12=k1(i)*k2(j)
        if (ck(k). gt. k12) go to 1300
        ck(k)=k12
1300    continue
c
        if (ii. gt. jj)go to 1400
        kk=jj-ii
        go to 1500
1400    kk=ii-jj
1500    continue
        if (ck(k). gt. k12)go to 1800
        ck(k)=k12
1800    continue
2000    continue
c
c  Read input friction numbers
c
        do 5000 l=1,nn
c
        do 2200 il=1,100
        mu(il)=0.0
2200    continue
c
        read *,x
c
c  Fuzzify the reading with composite kernel
c  and obtain L and R components.
c
        do 2500 k=1,200
        if (ck(k). le. 0.0)go to 2600
```

```
      kk1=2*k-1
      kk2=2*k
      y(kk1)=x+rint*(k-1)*x
      y(kk2)=x-rint*(k-1)*x
2500  continue
c
2600  continue
      kmax=k
      ymax=x+rint*(kmax-1)*x
      ymin=x-rint*(kmax-1)*x
c
c  Form a friction scale of 1-100 with intervals of 1.0
c
      do 4000 il=1,100
      z(il)=101-il
      if (z(il). gt. ymax) go to 4000
      if (z(il). lt. ymin) go to 4100
      do 3600 k=1,kmax
      kk1=2*k-1
      kk2=2*k
      test=rint*x*0.5
      if (abs(z(il)-y(kk1)). gt. test)go to 3000
      mu(il)=ck(k)
      go to 4000
3000  if (abs(z(il)-y(kk2)). gt. test)go to 3600
      mu(il)=ck(k)
      go to 4000
3600  continue
4000  continue
4100  continue
c
c  Write fuzzified friction number
c
      write(6,7500) 1
      write(6,7000)
      do 4500 il=1,100
      z(il)=101-il
      if (mu(il). lt. 0.01)go to 4500
      write(6,8000) z(il),mu(il)
4500  continue
c
5000  continue
c
c
7000  format(4x,'FN',7x,'membership',/)
7500  format(//,'pavement section no :',i3,//)
8000  format(2x,f6.3,7x,f6.3)
c
c
      stop
      end
```

```
c  PROGRAM DEF
c  This program can be used to :
c              i. fuzzify deflections for dynaflect imprecision.
c             ii. convert center deflections to edge deflections using
c                 fuzzy factors.
c            iii. convert fall deflections to spring deflections using
c                 fuzzy factors.
c
      implicit real(a-z)
      dimension x(50),kd(50),rrfd(100)
      dimension must(105),mus(50),z(105)
      integer i,j,k,n,nn,jl,j2,itr,il
c
c  input of the number of deflections and discretization intervals.
c
      read *,nn,n
c
c  input of ranges of fuzzification and fuzzy factors
c
      read *,rl,fl,f2,rfl,rf2
c
      an=n-1
c
      do 10 il=1,100
      must(il)=0.0
10    continue
c
c
      do 5000 i=1,nn
      write(6,8000) i
      r=rl
      int=r/an
      itr=1
c
c  input of deflection reading
c
      read *,def
c
c  fuzzify reading for imprecision
c
      do 100 j=1,n
      xx=(j-1)*int
      jl=2*j-1
      j2=2*j
      x(jl)=def+xx*def
      x(j2)=def-xx*def
c
      if (xx. gt. 0.5*r) go to 50
      kd(j)=1-2*((xx/r)**2)
       go to 100
50     kd(j)=2*(((xx-r)/r)**2)
100    continue
c
```

```
c  setting for multiplication by f1
c
        rf=rf1
        f=f1
        if (itr. eq. 1)go to 400
c
c  setting for multiplication by f2
c
200     continue
        itr=itr+1
        f=f2
        rf=rf2
400     continue
c
c  fuzzy multiplication
c
        fmin=f-rf*f
        fmax=f+rf*f
c
        fd=f*def
        rfd=(f+rf*f)*(def+r*def)
        rr=(rfd-fd)/fd
        int=rr/an
c
        do 2000 k=1,n
        mus(k)=0.00
        rrfd(k)=fd+int*fd*(k-1)
c
        do 1500 j=1,n
c
        j1=2*j-1
        j2=2*j
        cf=rrfd(k)/x(j1)
        ccf=rrfd(k)/x(j2)
c
c  obtain memberships of the right half
c
        if (cf. lt. fmin)go to 700
        if (cf. gt. fmax)go to 700
        test=abs(cf-f)
        if (test. gt. 0.5*rf*f)go to 600
        muf1=1-2*((test/rf/f)**2)
        go to 800
600     muf1=2*(((test-rf*f)/rf/f)**2)
c7
        go to 800
700     muf1=0.0
800     continue
c
c  obtain memberships of the left half
c
        if (ccf. lt. fmin)go to 907
        if (ccf. gt. fmax)go to 907
```

```
        test=abs(ccf-f)
        if (test. gt. 0.5*rf*f)go to 900
         muf2=1-2*((test/rf/f)**2)
        go to 905
900     muf2=2*(((test-rf*f)/rf/f)**2)
        go to 905
907     muf2=0.0
c
905     continue
c
        if (muf1. gt. muf2) go to 910
        muf=muf2
        go to 920
910     muf=muf1
920     continue
c
        if (kd(j). ge. muf) go to 1300
        mu=kd(j)
        go to 1350
1300    mu=muf
1350    continue
c
        if (mus(k). gt. mu)go to 1400
        mus(k)=mu
1400    continue
c
1500    continue
c
2000    continue
c
        if (itr. eq. 2)go to 3000
c
c  set initial conditions for second multiplication
c
        def=fd
        r=rr
c
        do 2500 j=1,n
        j1=2*j-1
        j2=2*j
        x(j1)=rrfd(j)
        x(j2)=2*fd-rrfd(j)
        kd(j)=mus(j)
2500    continue
c
       go to 200
c
3000    continue
c
        write(6,7000)
c                                                                    -4
c  bring deflection values to a common base of 1-100 (in.x 10    )
c
```

```
      do 4000 il=1,100
      rrmax=rfd
      rrmin=2*fd-rfd
c
      z(il)=101-il
      if (z(il). gt. rrmax)go to 4000
      if (z(il). lt. rrmin)go to 4100
c
      do 3600 k=1,n
      a=rrfd(k)
      b=2*fd-rrfd(k)
c
      test= int*fd*0.5
      if (abs(z(il)-a). gt. test)go to 3200
      must(il)=mus(k)
      go to 3700
3200  if (abs(z(il)-b). gt. test)go to 3600
      must(il)=mus(k)
      go to 3700
3600  continue
3700  continue
c
      write(6,6000)z(il),must(il)
4000  continue
c
4100  continue
c
5000  continue
c
6000  format(2x,f8.3,12x,f6.3)
7000  format('defltn.(10-4 in.)',3x,'membership',/)
8000  format(///,'pavement section no.',i3,//)
      stop
      end
```

```
c  PROGRAM DIST
c  This program is used to :
c      i. fuzzify relevant distress ratings for imprecision
c         associated with the extent measurement.
c     ii. fuzzify relevant distress ratings for the subjectivity
c         involved in severity determination.
c    iii. manipulate a fuzzy PCR for each pavement section.
c
c         PCR = 100 - sum ( distress ratings )
c     (* Memberships of supports exceeding 100.0 are truncated.)
c
c
c
       implicit real(a-z)
       dimension r1(13),r2(13),k1(50),k2(50),y(205),z1(105)
       dimension ck(205,13),z(105,13),mu(105,13),mus(105,13)
       integer i,j,k,l,m,n,ii,jj,kk1,kk2,jjj,il,nd,np,icount
c
c  Read the number of distress types and number of intervals
c
       read *,nd,n
       an=n-1
c
c  Read the ranges of imprecision and subjectivity
c        (Note: 1. if distress type has neither imprecision
c                  nor subjectivity use zeros.
c               2. if distress type has only one of the above
c                  treat it as r1.
c
       do 100 i=1,nd
       read *,r1(i),r2(i)
100    continue
c
c  Initialization
c
       do 150 j=1,n
       k1(j)=0.0
150    continue
c
       do 200 k=1,200
       y(k)=0.0
       do 180 j=1,nd
       ck(k,j)=0.0
180    continue
200    continue
c
       do 300 il=1,100
       z1(il)=0.0
       do 250 j=1,nd
       z(il,j)=0.0
250    continue
300    continue
c
```

```
c  Assigning of membership values to the kernels.
c
       do 2200 j=1,nd
       int=r1(j)/an
       do 500 i=1,n
       rr1=(i-1)*int
       if (r1(j). ne. 0.0)go to 350
       k1(i)=1.0
       go to 600
350     if (rr1. gt. 0.5*r1(j))go to 400
       k1(i)=1-2*((rr1/r1(j))**2)
       go to 500
400    k1(i)=2*(((rr1-r1(j))/r1(j))**2)
500    continue
c
600    continue
       do 1000 jj=1,50
       rr2=(jj-1)*int
       if (r2(j). ne. 0.0)go to 750
       k2(jj)=1.0
       go to 1100
750    if (rr2. gt. r2(j))go to 1100
       if (rr2. gt. 0.5*r2(j))go to 800
       k2(jj)=1-2*((rr2/r2(j))**2)
       go to 1000
800    k2(jj)=2*(((rr2-r2(j))/r2(j))**2)
1000   continue
c
1100   m=jj
       do 2000 i=1,n
       ii=i-1
       do 1800 jj=1,m
       jjj=jj-1
       kk=ii+jjj
       k=kk+1
       k12=k1(i)*k2(jj)
       if (ck(k,j). gt. k12) go to 1300
       ck(k,j)=k12
1300   continue
c
       if (ii. gt. jjj)go to 1400
       kk=jjj-ii
       go to 1500
1400   kk=ii-jjj
1500   continue
       if (ck(k,j). gt. k12)go to 1800
       ck(k,j)=k12
1800   continue
2000   continue
2200   continue
c
       icount=0
```

```
c  Read the number of pavement sections
c
      read *,np
2300  continue
c
      do 2400 il=1,100
      do 2350 j=1,nd
      mu(il,j)=0.0
      mus(il,j)=0.0
2350  continue
2400  continue
c
      icount=icount+1
      write(6,8000)icount
c
      do 5000 j=1,nd
      read *,x
c
      do 2500 k=1,200
      if (ck(k,j). le. 0.0)go to 2600
      kk1=2*k-1
      kk2=2*k
      y(kk1)=x+int*(k-1)*x
      y(kk2)=x-int*(k-1)*x
2500  continue
c
2600  continue
      kmax=k
      ymax=x+int*(kmax-1)*x
      ymin=x-int*(kmax-1)*x
c
      do 4000 il=1,101
      z(il,j)=101-il
      if (z(il,j). gt. ymax)go to 4000
      if (z(il,j). lt. ymin)go to 4100
      do 3600 k=1,kmax
      kk1=2*k-1
      kk2=2*k
      test=int*x*0.5
      if (abs(z(il,j)-y(kk1)). gt. test)go to 3000
      mu(il,j)=ck(k,j)
      go to 4000
3000  if (abs(z(il,j)-y(kk2)). gt. test)go to 3600
      mu(il,j)=ck(k,j)
      go to 4000
3600  continue
4000  continue
4100  continue
c
      do 4500 i=1,101
      z1(i)=101-i
      do 4400 k=1,101
```

```
      if (mu(k,j). lt. 0.001)go to 4400
      if (j. gt. 1)go to 4150
      mus(k,j)=mu(k,j)
      go to 4400
4150  diff=z1(i)-z(k,j)
      if (diff. lt. 0.0)go to 4400
c
      do 4300 l=1,101
      zp=101-l
      jj=j-1
      if (mus(1,jj). lt. 0.001)go to 4300
      if (diff. ne. zp)go to 4300
      if (mu(k,j). gt. mus(1,jj))go to 4200
      mup=mu(k,j)
      go to 4210
4200  mup=mus(1,jj)
4210  if (mus(i,j). gt. mup)go to 4250
      mus(i,j)=mup
4250  continue
      go to 4400
4300  continue
4400  continue
      if (j. eq. 1)go to 4600
4500  continue
4600  continue
c
5000  continue
c
      write (6,7000)
      do 6000 i=1,101
      z1(i)=i-1
      if (mus(i,nd). lt. 0.001)go to 6000
      write(6,7500)z1(i),mus(i,nd)
6000  continue
c
      if (icount. lt. np)go to 2300
c
7000  format(5x,'PCR',8x,'membership',/)
7500  format(2x,f6.2,10x,f6.3)
8000  format(///,'pavement section no.',i5,//)
c
      stop
      end
```

```
c  PROGRAM DM
c  This program is used to ;
c          1. obtain the fuzzy priorities from attribute values
c          2. rank the fuzzy priorities
c
       Implicit real (a-z)
       dimension nav(3),mu(1000),mumax(105),mus(50,105)
       dimension suput(50),ut(105),bb(50,5),db(50,5),mut(1000,105)
       dimension at(3,50),aat(3),atmu(3,50),uut(5),duut(5)
       integer i,j,k,k1,k2,k3,l,n,m,ii,nt,nav,ne
c
c  Read the number of pavement sections
c
       read *,n
c
c  Read attribute values
c
       do 5000 i=1,n
c
       do 200 j=1,3
c
       read *,nav(j)
c
       do 100 k=1,nav(j)
       read *, at(j,k),atmu(j,k)
100    continue
c
200    continue
c
c  Form attribute tuples
c
       nt=nav(1)*nav(2)*nav(3)
c
       ii=1
c
       do 1000 k1=1,nav(1)
       mu(ii)=atmu(1,k1)
       do 800 k2=1,nav(2)
       if (k2. eq. 1) dum2=mu(ii)
       if (k2. gt. 1) mu(ii)=dum2
       if (mu(ii). lt. atmu(2,k2)) go to 500
       mu(ii)=atmu(2,k2)
500    continue
       do 600 k3=1,nav(3)
       if (k3. eq. 1) dum1=mu(ii)
       if (k3. gt. 1) mu(ii)=dum1
       if (mu(ii). lt. atmu(3,k3)) go to 550
       mu(ii)=atmu(3,k3)
550    continue
c
       ii=ii+1
600    continue
800    continue
```

```
1000    continue
c
c  Obtain the corresponding utilities
c
        suput(i)=1.0
c
        ii=1
c
        do 2000 k1=1,nav(1)
        do 1800 k2=1,nav(2)
        do 1600 k3=1,nav(3)
c
        aat(1)=at(1,k1)
        aat(2)=at(2,k2)
        aat(3)=at(3,k3)
c
        call interpol(ii,uut,duut,ne,aat,i,db,bb,m)
c
c  obtain the fuzzy utilities
c
        call fuzzut(uut,duut,ii,ne,ut,mut)
c
c
        ii=ii+1
1600    continue
1800    continue
2000    continue
c
c
        do 2400 l=1,91
c
c  Initialize
c
        mus(i,l)=0.0
c
c  Obtain a fuzzy priority value for the section
c
        do 2300 ii=1,nt
c
        if (mut(ii,l). eq. 0.0) go to 2100
        if (suput(i). gt. ut(l)) go to 2100
c
        suput(i)=ut(l)
2100    continue
c
        if (mut(ii,l). lt. mu(ii)) go to 2200
        dum=mu(ii)
        go to 2250
2200    dum=mut(ii,l)
2250    continue
c
        if (mus(i,l). gt. dum) go to 2300
        mus(i,l)=dum
```

```
2300    continue
2400    continue
c
5000    continue
c
c  Form the maximizing set
c
        supp=0.0
c
        do 6000 i=1,n
        if (supp. gt. suput(i)) go to 6000
        supp=suput(i)
6000    continue
c
        do 7000 1=1,91
        if (ut(1). gt. supp) go to 7100
        mumax(1)=ut(1)/supp
7000    continue
7100    continue
c
c  Calculate a relative rank for the section
c
        write(6,8400)
8400    format('section',3x,'rel. priority',///)
c
        do 9500 i=1,n
c
        opt=0.0
c
        do 9000 1=1,91
        if (mus(i,1). 1t. mumax(1)) go to 8500
        mus(i,1)=mumax(1)
8500    continue
        if (opt. gt. mus(i,1)) go to 9000
        opt =mus(i,1)
c
9000    continue
c
        write (6,9100)i,opt
9100    format(2x,i5,x,f10.3,/)
c
9500    continue
c
        stop
        end
c
c
c
        subroutine interpol(ii,uut,duut,ne,aat,i,db,bb,n)
c
c  Given the attribute values, this subroutine calculates
c  the corresponding relative priority of the section.
```

```
      implicit real (a-z)
      dimension aat(3),utst(5,50),wkarea(50),dutst(5,50),bb(50,5)
      dimension atst(3,50),a(50,50),b(50,1),db(50,5),uut(5),duut(5)
      integer ii,i,nn,n,ne,k,kk,j,jj,l,ll,m,ia,idgt,ier
c
c  Read utilities provided by experts and the corresponding
c  attribute values.
c
      if (ii. gt. 1) go to 130
      if (i. gt. 1) go to 130
c
      read *,n,ne
c
      nn=3*n+1
      m=1
      idgt=0
      ia=50
c
      do 100 k=1,nn
      read *,atst(1,k),atst(2,k),atst(3,k)
100   continue
c
      do 125 k=1,nn
      do 110 l=1,ne
      read *,utst(l,k),dutst(l,k)
110   continue
125   continue
130   continue
c
c
      do 1800 l=1,ne
c
      ll=l
150   continue
      if (ii. gt. 1) go to 1100
      if (i. gt. 1) go to 1100
c
      do 300 k=1,nn
c
      if (ll. eq. 1) b(k,1)=utst(1,k)
      if (ll. eq. 2) b(k,1)=dutst(1,k)
      a(k,1)=1.0
c
      do 200 j=1,n
c
      jj=3*j-1
      a(k,jj)=atst(1,k)**j
      a(k,jj+1)=atst(2,k)**j
      a(k,jj+2)=1.0/atst(3,k)**j
c
200   continue
300   continue
```

```
c  Solve the set of 3n+1 simultaneous equations
c  and store the solution in b vector.
c
      call leqtlf(a,m,nn,ia,b,idgt,wkarea,ier)
c
c  Find the utility corresponding to the attributes
c  given in the main program
c
c
      do 1000 k=1,nn
      if (ll. eq. 1) bb(k,1)=b(k,1)
      if (ll. eq. 2) db(k,1)=b(k,1)
1000  continue
1100  continue
c
      if (ll. eq. 1) aut=bb(1,1)
      if (ll. eq. 2) autl=db(1,1)
c
      do 1300 k=1,n
      kk=3*k-1
c
      aut=aut+bb(kk,1)*aat(1)**k
      aut=aut+bb(kk+1,1)*aat(2)**k
      aut=aut+bb(kk+2,1)/aat(3)**k
      autl=autl+db(kk,1)*aat(1)**k
      autl=autl+db(kk+1,1)*aat(2)**k
      autl=autl+db(kk+2,1)/aat(3)**k
1300  continue
c
      if (ll.eq. 2) go to 1500
      uut(1)=aut
      go to 1600
1500  duut(1)=autl
c
1600  continue
      if (ll. eq. 2) go to 1800
      ll=2
      go to 150
1800  continue
c
      return
      end
c
c
c
      subroutine fuzzut(uut,duut,ii,ne,ut,mut)
c
c  This subroutine calculates the fuzzy utilities, given
c  the assigned utilities and the ranges of each expert.
c
      implicit real (a-z)
      dimension uut(5),duut(5),ut(105),mut(1000,105)
```

```
      integer ii,ne,ll,l
c
c  Initialization
c
      do 100 ll=1,91
      mut(ii,ll)=0.0
100   continue
c
      do 1000 l=1,ne
      do 500 ll=1,91
c
c  Use a standard interval for utilities
c
      ut(ll)=1.0+(ll-1)*0.1
      if (ut(ll). lt. uut(l)) go to 500
      if (ut(ll). gt. (uut(l)+duut(l))) go to 500
c
      mut(ii,ll)=mut(ii,ll)+1
c
500   continue
1000  continue
c
      do 1500 ll=1,91
      if (mut(ii,ll). eq. 0.0) go to 1500
      mut(ii,ll)=mut(ii,ll)/ne
1500  continue
c
      return
      end
```

```
c  PROGRAM REGRESS
c  this program can be used to :
c       i.  obtain the fuzzy coefficients of the linear regression
c           equation between the roadmeter reading and PSR.
c      ii.  to calculate the fuzzy PSR for a given roadmeter reading.
c
c
       implicit real(a-z)
       dimension rr(30),mu(50,30)
       integer i,j,k,l,m,n,ii,nn
c
c  read the input data : roadmeter reading and PSR
c
       read *,n,H
       write(6,370)
       write(6,380)n,H
c
       do 5 j=1,n
       read *, rr(j)
          do 2 i=1,50
          read *, mu(i,j)
2         continue
5      continue
c
c  read the initial values of the parameters:
c
       read *, alph1,alph2,delph1,delph2,delc1,delc2
       write(6,385)
       write(6,390)alph1,alph2
c
       d=1.0
c
       do 1000 nn=1,20
       a=1.0
       alpha1=alph1+(nn-1)*delph1
       go to 20
c
10     a=-1.0
       alpha1=alph1-(nn-1)*delph1
c
20     continue
c
          do 800 m=1,10
          b=1.0
          alpha2=alph2+(m-1)*delph2
          go to 40
c
30        b=-1.0
          alpha2=alph2-(m-1)*delph2
c
40        continue
c
             do 600 l=1,50
```

```
            c1=(1-1)*delc1
            if (c1. gt. d)go to 700
c
c
               do 400 k=1,20
               c2=(k-1)*delc2
               if (c2. gt. d)go to 700
c
c
                  do 300 j=1,n
c
                     do 50 i=1,50
                     if (mu(i,j). eq. 0.0) go to 50
                     minx=i
                     go to 60
50                   continue
c
60                 continue
c
                     do 80 i=1,50
                     ii=50-i
                     if (mu(ii,j). eq. 0.0) go to 80
                     maxx=ii
                     go to 90
80                   continue
c
90                 continue
c
                   alpha=10.0*(alpha1+alpha2*rr(j))
                   c=10.0*(c1+c2*rr(j))
c
                   maxs=alpha+c
                   mins=alpha-c
c
                   if (maxx. gt. maxs) go to 400
                   if (minx. lt. mins) go to 400
c
c
                     do 150 i=1,50
                     if (i. lt. minx)go to 150
                     if (i. gt. alpha)go to 160
c
                     h1=1-(alpha-i)/c
                     diff1=h1-mu(i,j)
c
                     if (diff1. lt. 0.05) go to 160
150                  continue
c
160                continue
c
                     do 250 i=1,50
                     ii=50-i
                     if (ii. gt. maxx) go to 250
```

```
                        if (ii. lt. alpha) go to 260
                        h2=1-(ii-alpha)/c
                        diff2=h2-mu(ii,j)
c
                        if (diff2. lt. 0.05)go to 260
250                     continue
c
260                  continue
c
                     if (h1. lt. h2)go to 280
                     h1=h2
280                  if (h1. ge. H) go to 300
                     go to 400
c
300                  continue
c
c  store the set giving the least c1+c2
c
                  if ( (c1+c2). gt. d) go to 700
                  d=c1+c2
                  alp1=alpha1
                  alp2=alpha2
                  cc1=c1
                  cc2=c2
                  go to 700
c
400               continue
c
600            continue
c
700         if (b. gt. 0.0) go to 30
800         continue
c
            if (a. gt. 0.0) go to 10
1000     continue
c
c  print the fuzzy parameters
c
                  write(6,350)
                  write(6,360)alp1,alp2,cc1,cc2
350               format(2x,'alpha1',5x,'alpha2',7x,'c1',9x,'c2',/)
360               format(2x,f4.2,5x,f8.6,5x,f4.2,5x,f9.7)
370               format('no. of sets',3x,'degree of fitting'/)
380               format(3x,i3,10x,f4.3,//)
385               format('trial alpha1',3x,'trial alpha2',/)
390               format(4x,f4.2,8x,f8.6,//)
         stop
         end
```

```
c  PROGRAM LIFE
c  This program is used to calculate the fuzzy PSI
c  life of a pavement.
c
c
       implicit real (a-z)
       dimension t(50),pr(15),up(15),ti(50),tj(50)
       dimension mupr(15),muup(15),muat(60)
       integer i,j,k,jj
c
c  Read input data
c    1. performance vs. time
c
       do 100 i=1,50
       read *, t(i)
100    continue
c
c    2. present performance level
c
       do 200 j=1,15
       read *, pr(j), mupr(j)
200    continue
c
c    3. unacceptable performance level
c
       do 300 j=1,15
       read *, up(j), muup(j)
300    continue
c
c  calculate times corresponding to present
c  and unacceptable levels
c
       do 1000 i=1,50
       p=4.2-0.1*i
c
       do 600 j=1,15
       if (pr(j). ne. p) go to 350
       ti(j)=t(i)
350    continue
c
       if (up(j). ne. p) go to 450
       tj(j)=t(i)
450    continue
1000   continue
c
c  calculate the fuzzy serviceability life
c
       do 2000 jj=1,15
       do 1500 j=1,15
c
       tk=tj(jj)-ti(j)
       if (tk. le. 0.0) go to 1500
c
```

```
          mu =muup(jj)
          if (mu. lt. mupr(j)) go to 1100
          mu = mupr(j)
1100      continue
c
          do 1200 k=1,60
          kk=k/10.0
          if (abs(tk-kk). gt. 0.05) go to 1200
          if (mu. lt. muat(k)) go to 1200
          muat(k)=mu
1200      continue
1500      continue
2000      continue
c
c  print the fuzzy serviceability life
c
          do 3000 k=1,60
          kk=k/10.0
          if (muat(k). lt. 0.01) go to 3000
          print(6,3100) kk,muat(k)
3000      continue
c
3100      format(2f8.3)
          end
          stop
```

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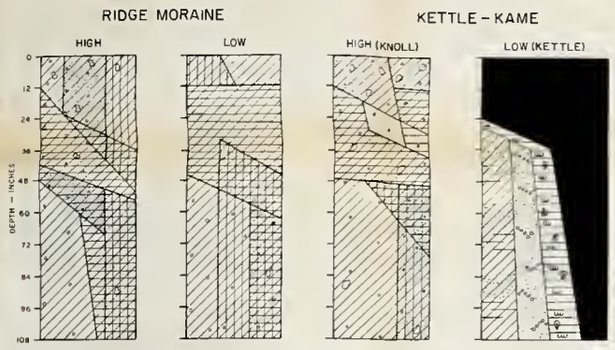
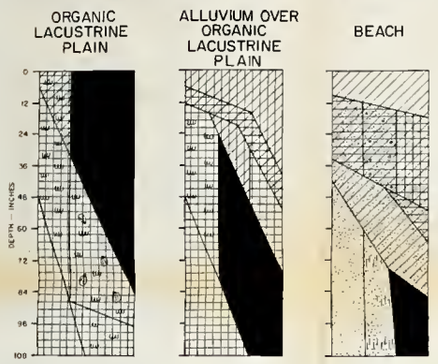
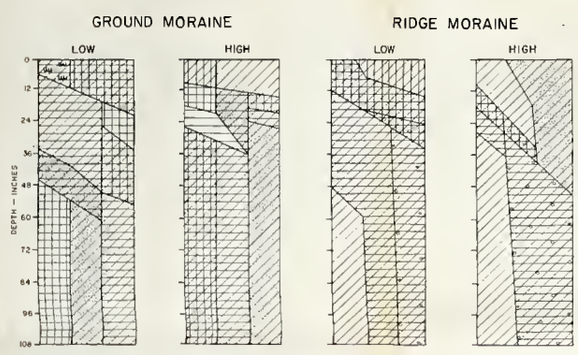
GENERAL SOIL PROFILES

PREDOMINANT SOILS NORTH OF THE EEL RIVER

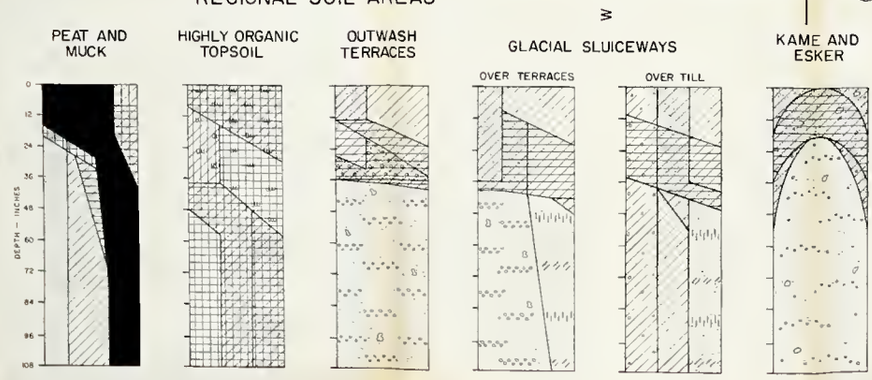


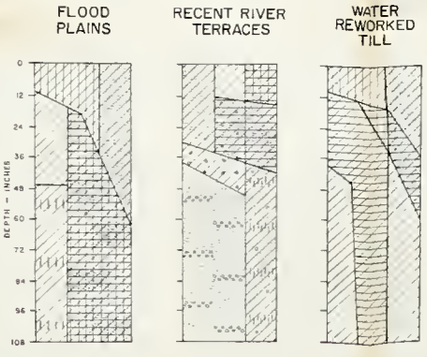


PREDOMINANT SOILS SOUTH OF THE EEL RIVER

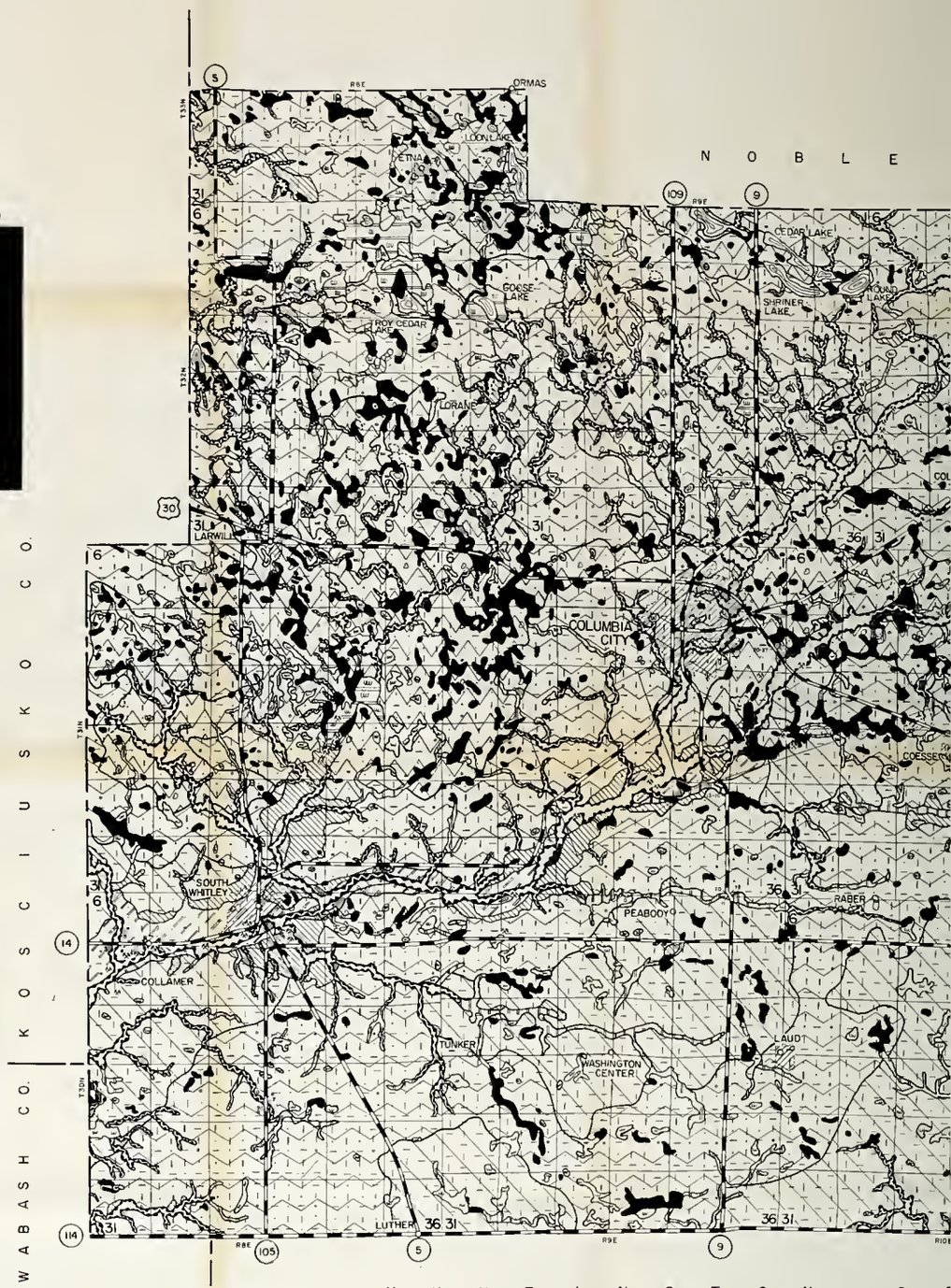


REGIONAL SOIL AREAS





TEXTURAL SYMBOLS FOR SOIL PROFILES



ENGINEERING SOILS MAP

WHITLEY COUNTY

INDIANA

PREPARED FROM

1951 A A A AERIAL PHOTOGRAPHS

BY

JOINT HIGHWAY RESEARCH PROJECT

AT

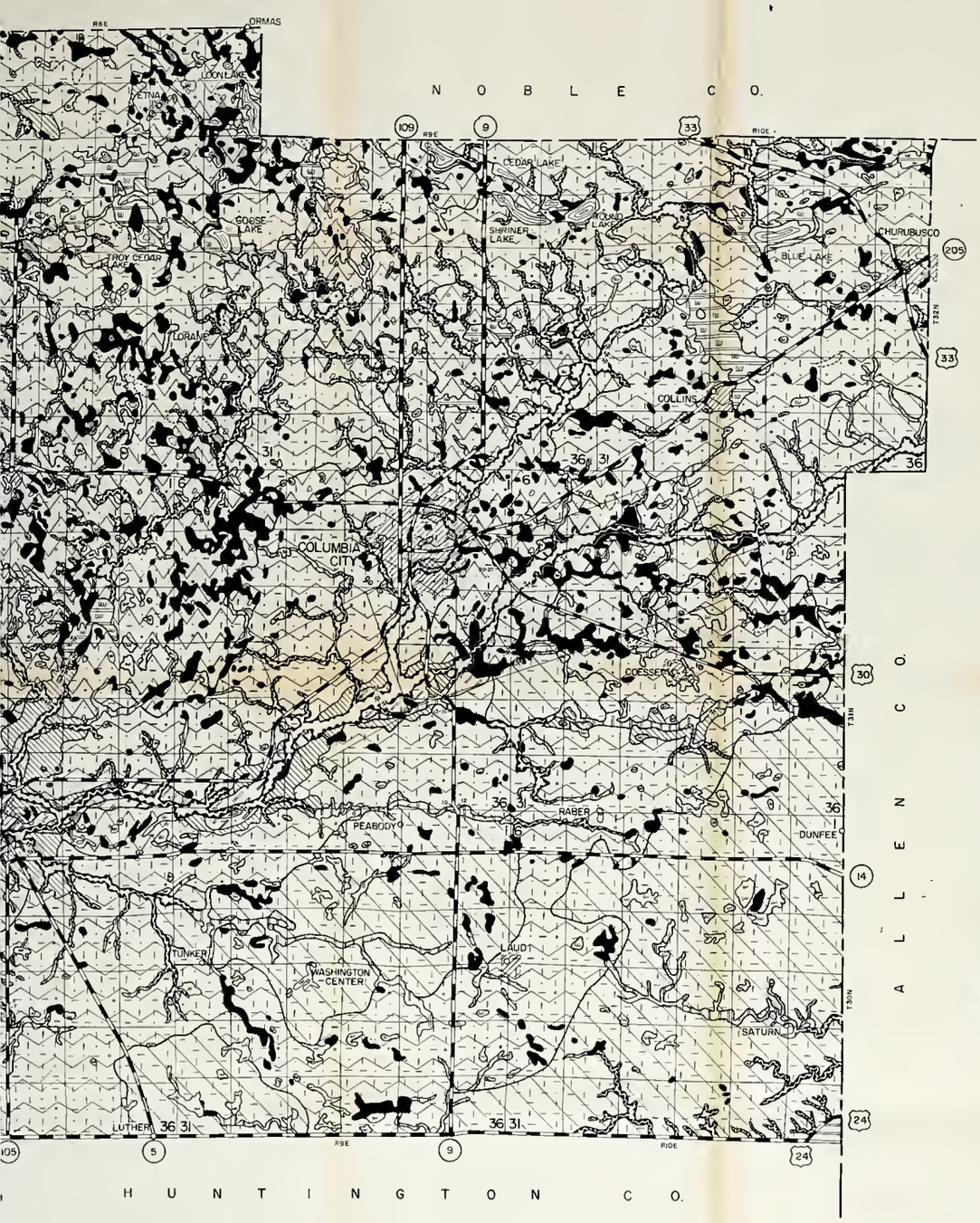
PURDUE UNIVERSITY

1984

SCALE OF MILES

1 0 1 2 3 4

PREPARED UNDER THE SUPERVISION OF E. M. GEFELL AND R. D. MILES



ENGINEERING SOILS MAP
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SCALE OF MILES

LEGEND

PARENT MATERIALS
(GROUPED ACCORDING TO LAND FORM AND ORIGIN)

- GROUND MORAINE
- RIDGE MORAINE
- KETTLE–KAME MORAINE
- OUTWASH TERRACE
- RECENT RIVER TERRACE
- BEACH
- FLOOD PLAIN
- GLACIAL SLUICEWAY
- WATER REWORKED TILL
- ORGANIC LACUSTRINE PLAIN
- KAME AND ESKER
- SLACKWATER LACUSTRINE PLAIN

MISCELLANEOUS

- LAKE OR POND
- SAND OR GRAVEL PIT
- BORING SITES
- URBAN AREAS

TEXTURAL SYMBOLS
(SUPERIMPOSED ON PARENT MATERIAL TO SHOW RELATIVE COMPOSITION)

- GRAVEL
- SAND
- COBBLE OR BOULDER
- PEAT AND MUCK
- SILT
- CLAY
- HIGHLY ORGANIC TOP SOIL

DRAWN BY D. W. GONZALES